Letter of Interest to AF4: Multi-TeV Collider and AF2: Accelerator for Neutrino Physics
Subject: Applications of Vertical Excursion FFAs (vFFA) and Novel Optics
From: Shinji Machida, on behalf of muon collider collaboration

Acceleration and accumulation of muon beams, whether for a neutrino physics facility or a multi-TeV muon collider, is a big challenge from the machine design point of view. The life time of muons in the rest frame is only 2.2 micro second. Acceleration has to be as fast as possible to prevent muon decay and increase survival up to the final energy. As tertiary produced particles, the muon beam emittance may not be as small as conventional accelerator beams such as protons or electrons.

The use of Fixed Field Alternating Gradient Accelerators (FFAs) for muon acceleration has been proposed as the main accelerator for a neutrino factory since the 1990s [1-4]. When it was invented, one of the big advantages of a FFA was a high operating repetition rate providing high average beam current [5-7]; this is limited in a synchrotron which requires ramping of the main lattice magnets. For muon acceleration, short duration of an acceleration cycle rather than a high repetition rate is essential.

R&D over the last 20 years has advanced our knowledge of FFAs in terms of beam optics and necessary hardware. New concepts in beam optics, such as a linear nonscaling FFA, a pumplet FFA, serpentine channel acceleration and a DF spiral FFA, have been proposed and some have been demonstrated experimentally [8]. However, for the fastest possible acceleration of muon beams, any change of the revolution frequency through an acceleration cycle is a bottleneck in order to increase the momentum range from injection to extraction. In both serpentine channel acceleration and acceleration within a stationary RF bucket, the momentum ratio is a few times at most.

A FFA with vertical orbit excursion (vFFA) was first proposed by Ohkawa in 1955 [9] and recently reinvented by Brooks in 2013 [10]. In a vFFA, the circular orbit moves up or down depending of the magnetic field shape when the beams are accelerated. As Ohkawa named it an ‘electron cyclotron’ in his paper, the momentum compaction factor of a vFFA is zero for the entire momentum range and the revolution frequency becomes independent of the particle momentum for ultra-relativistic particles. This opens up a new way for accelerating unstable particles like muons.

For a muon collider with a final energy of 3 TeV, an order of 100 to 1000 times acceleration in terms of the momentum is required and we expect this to be done in circular ring accelerators. Although it might be unavoidable to have multi-stage ring accelerators, the large momentum ratio between injection and extraction reduces the number of stages and the total cost. vFFAs can use a fixed frequency RF system and this gives two major advantages. First the energy gain per turn could be high because of the fixed frequency RF and the beams are always near the crest of the RF wave form. Secondly the momentum ratio is only determined by the lattice magnet configuration, not by the longitudinal acceptance. If we could choose a large field index and allow a reasonable orbit excursion, e.g. half a metre, a momentum ratio of 30 is achievable [11], which affords the scenario of two stage muon accelerators from the GeV to the TeV level.

A collider ring does not require a large momentum acceptance, but some of the characteristics of a vFFA could be beneficial. For example, wriggling orbits both in horizontal and vertical planes help to reduce the convergence of the neutrino beams to certain directions. Slight adjustments of the optics will change the wriggling pattern of the orbits so that we can control the direction of the neutrino beams. The isochronous condition would keep the bunch shape fixed, which is essential to maintain high luminosity.

A study has been initiated to design a collider ring extracting essential ingredients of the vFFA. For example, arc section optics comprising skew quadrupoles (which is the lowest multipole of the vFFA magnets) have been investigated [12]. A skew quadrupole lattice cannot provide isochronous conditions for a fully wide range of momentum, but can make the zeroth order momentum compaction factor (alpha_c) zero. In contrast to the common method employed in a normal quadrupole lattice to make alpha_c zero, a lattice with skew quadrupoles does not need negative bending magnets. Therefore the peak magnetic fields become lower or the circumference of a ring can be smaller.

The muon accumulator ring of the LEMMA scheme [13] has similar requirements to the muon collider ring in terms of optics design, aiming at a compact ring and an isochronous condition over a momentum range of +/-10 to 20%. Studies of an arc with skew quadrupoles already indicates a promising initial design. Both collider ring and accumulator ring have low beta insertions. It would be natural to consider the straight line optics with skew quadrupoles so that the matching between the arc and the low beta insertion could be done smoothly. Wriggling orbits in both vertical and horizontal planes in the straight section are advantageous to enhance divergence of neutrino flux. The whole ring with skew quadrupoles could be converted to a normal quadrupole lattice when the reference coordinates are rotated by 45 degrees. In that rotated frame, weak focusing at the bending of the arc becomes skew so that a correction skew quadrupole (which would be a standard quadrupole in the normal horizontal and vertical frame) may be necessary.

The large acceptance of FFAs in general, not only of the vFFA, could be ideal for the positron ring of the LEMMA scheme where the positron beams spread out when they go through the target. The proton driver of the proton-based muon collider scheme is another area where a FFA could be employed.

Despite its potential as an ideal accelerator for the muon collider complex, there are several issues requiring further studies. As an accelerator, the present vFFA design has relatively strong reverse bending magnets. The circumference factor, which is defined roughly as the ratio of ring circumference to that of a ring with only bending magnets, is large, at least 5 in the latest design. This could be further reduced by a proper choice of optics parameters, e.g. asymmetrical tunes in the two orthogonal planes. For the collider and accumulator ring, it is also important to minimise the ring size to reduce the cost. The skew quadrupole-based lattice design for both arc and low beta insertions may require development of unconventional algorithms for orbit and optics corrections. Superconducting magnets which give the correct shape of magnetic field profile and strength have to be designed. These are just a few examples.

Much of the experience of the current team members was gained through the design, construction and commissioning of EMMA (Electron Model of Muon Accelerator) in 2012 [14], which is the first linear nonscaling FFA to demonstrate serpentine channel acceleration and resonance crossing with fast acceleration. The team is also engaged in designing a vFFA as a prototype for a future accelerator upgrade of the ISIS spallation neutron and muon source, named ISIS-II [15] at the Rutherford Appleton Laboratory. We expect much progress on the vFFA development beyond purely theoretical studies in the coming years.

This work will be pursued within the newly formed International Muon Collider collaboration.

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