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Phase Transitions: Precision Calculations of Gravitational Wave Spectrum and Thermal Parameters

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■ (EF02) EW Physics: Higgs Boson as a portal to new physics
■ (CF7) Cosmic Probes of Fundamental Physics
■ (EF01) EW Physics: Higgs Boson properties and couplings

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Abstract: The purpose of this LOI is to study phase transitions from two complementary aspects. (*i*) Firstly, we propose to perform precision calculations of the stochastic gravitational wave spectrum with the goal of obtaining accurate benchmarks for complementarity studies with future colliders. The goal is also to possibly uncover non-standard cosmological histories; and (*ii*) Secondly, we aim to perform precision calculations of the thermal parameters. This is motivated by the fact that the overall peak amplitude of the gravitational wave spectrum typically scales with the square of the trace anomaly and the square of the time of the transition; this dependency leads to a dramatic amplification of the theoretical uncertainties of the physical observables generated from a given BSM model.

1 Precision Calculation of Gravitational Wave spectrum

Ref. [1] initiated a detailed analysis of stochastic gravitational wave production from cosmological phase transitions in an expanding universe, studying both a standard radiation as well as a matter dominated history. A detailed analysis was performed for the dynamics of the phase transition including the false vacuum fraction, bubble lifetime distribution, bubble number density, mean bubble separation, etc., for an expanding universe. The authors also studied the full set of differential equations governing the evolution of plasma and the scalar field during the phase transition and generalized results obtained in Minkowski spacetime. In particular, the sound shell model was generalized to the expanding universe and an accurate calculation of the gravitational wave spectrum seen today for the dominant source of sound waves was provided.

For the amplitude of the gravitational wave spectrum visible today, a suppression factor arising from the finite lifetime of the sound waves was found. This was compared with the commonly used result in the literature, which corresponds to the asymptotic value of the suppression factor. The asymptotic value is only applicable for a very long lifetime of the sound waves, which is highly unlikely due to the onset of shocks, turbulence and other damping processes.

The next step is to apply this calculation of gravitational waves in an expanding Universe to various models, such as SMEFT, xSM, and models of confining hidden sectors, to obtain accurate benchmark predictions for the gravitational wave spectrum. This will involve further analytical development of the sound shell model. An allied direction is to utilize phase transitions as cosmic witnesses, that is, to use the associated spectrum to distinguish between *different* expansion histories. The change in the spectrum is not leading order, due to the fact that the velocity profiles remain largely unchanged and that the autocorrelation time of the source is much smaller than the duration of the transition. This is in contrast to gravitational waves generated from cosmic strings. Even then, the modification of the spectrum presents an enticing possibility that the gravitational waves formed during a phase transition can bear witness to an early matter dominated era.

2 Precision calculation of thermal parameters

Calculation of thermal parameters requires a consistent treatment of the effective potential at finite temperature. Current state of the art perturbative treatments suffer from large theoretical uncertainties from three dominant sources

- The expansion parameter at finite temperature is gT/m for some coupling g [2]. This means that perturbativity breaks down at finite temperature in both phases - the gauge bosons suffer infrared divergences in the symmetry phase and the Goldstone modes in the broken phase. Current resummation methods are ad-hoc and fail to bring the uncertainties from scale dependence under control [3]
- The contribution of Goldstone modes to the effective potential leads to a gauge dependence when performing a loop expansion. One can perform a different expansion (the $\hbar$ expansion [4]) to remove the gauge dependence at the cost of making the scale dependence even worse than it is in the loop expansion (comparing first order in both) [5]
- The effective potential is calculated assuming a homogeneous background. This calculation is then used to calculate the features of the inhomogenious background.

Such theoretical uncertainties can appear modest when calculating critical or nucleation temperatures, as the relative uncertainty is often in the $10-50\%$ range [3]. The trace anomaly naively has a quartic dependence on the temperature and the inverse timescale has naively a linear dependence. The overall peak amplitude of the gravitational wave spectrum resulting from a phase transition typically scales with the square of the trace anomaly and the square of the time of the transition. This dependency leads to a dramatic amplification of the theoretical uncertainties of the physical observables generated from a given BSM model. In fact the relative uncertainty is frequently orders of magnitude! This depressing reality about the current state of the art dampens the exciting prospect of gravitational wave physics being complimentary with other BSM seachers (especially collider probes).

One approach that seems to manage all three dominant theoretical uncertainties is dimensional reduction, where heavy and super heavy matsubara modes are integrated out [6]. The resulting theory is manifestly gauge invariant and the generated resummation seems to effectively control infrared divergences. Remarkably, alternatives have been developed to the gradient expansion in this framework which means even the third error listed above can be resolved within this framework. The downside of dimensional reduction is it becomes quite cumbersome and awkward when the static mode is heavy or when there are multiple dynamical modes. Both speedhumps are common in BSM models. It is therefore attractive to theoretically advance perturbative techniques.

We therefore propose to move in two directions - improving technology of dimensional reduction to apply to more challenging BSM scenarios, and secondly to theoretically improve perturbative techniques to reduce the theoretical uncertainties. Recent work by Curtin *et al* [7] has proposed a more consistent approach to resummation of scalar modes. Such techniques can be extended to the gauge boson modes and assessed for their handling of the scale dependence. Second, the treatment of the inhomogeneous background has recently been performed self consistently at zero temperature by Bjorn Garbrecht et al [8, 9]. Applying such techniques to the finite temperature case should eliminate this theoretical uncertainty.

3 Collider Complimentarity

One of the main theoretical questions that future colliders can answer is "what is the phase diagram of electroweak symmetry" [10]. Recent work has not included theoretical uncertainty or accounted for the fact that different channels can be complimentary to each other. In particular, the branching ratio of singlet to vector bosons tends to get larger when its branching ratio to Higgs pairs becomes small (and vice versa). A careful treatment of the theoretical uncertainty as well as a multiple channel analysis is needed to make definitive statements about what specifications are needed for a collider to test the order of the electroweak phase transition. Complementary studies between gravitational waves and di-Higgs collider studies has been performed in the context of the real scalar singlet extended Standard Model Higgs sector (xSM model) in [11–14].

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