Lattice Calculation of Neutrino-Nucleon Cross Section

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On behalf of the χ QCD Collaboration (a part of the the USQCD Collaboration), we submit this Letter of Interest (LOI) to disclose our plan to carry out lattice calculation of the neutrino-nucleon scattering cross section for the neutrino energy less than $\sim$ 7 GeV, in the range of the DUNE experiments. The relevance of lattice calculations to neutrino-nucleon scattering is summarized in an USQCD whitepaper [1]. Here we discuss the calculation of the hadronic tensor to cover the neutrino energy to $\sim$ 7 GeV.

Lepton-nucleon scattering cross-section is a product of the leptonic tensor and the hadronic tensor. The hsdronic tensor is the imaginary part of the forward amplitude, and can be expressed as the current-current correlation function in the nucleon, i.e.

$$W_{\mu\nu}(q^2,\nu) = \frac{1}{\pi}\mathrm{Im}\, T_{\mu\nu}(q^2,\nu) = \int \frac{d^4x}{4\pi} e^{iq\cdot x}\langle N|J_\mu(x)J_\nu(0)|N\rangle_{\text{spin ave.}}. \tag{1}$$

The expected spectral density of the neutrino-nucleon scattering cross-section or structure functions shows that there are several kinematic regions in the spectral density in the energy transfer ν – the elastic scattering, the inelastic reactions ($\pi N, \pi\pi N, \eta N$ etc.) and resonances (Δ, Roper, S_{11}, etc.), the shallow inelastic scattering (SIS), and the deep inelastic scattering (DIS) regions.

For scatterings at lower energies, the nucleon hadronic tenor is needed, together with many-body calculation of the nucleus, to delineate the experiments of neutrino-nucleus scattering, e.g. LBNF/DUNE [2] at Fermilab, which aims to study the neutrino properties. The beam energy of DUNE is in the range $\sim$ 1 to $\sim$ 7 GeV and at different beam energies, different contributions (quasi-elastic (QE), resonance (RES), SIS and DIS) will dominate the total cross section [3]. The hadronic tensor is useful in the whole energy region. For example in the QE region, the hadronic tensor is actually the square of the elastic form factors of nucleon and the cross-section of neutrino-nucleus scattering can be calculated by combining the nucleon form factors and nuclear models about the nucleon distribution inside a nucleus. In the RES, SIS and DIS region, inelastic neutrino-nucleon scatterings emerge and one will need to have inclusive hadronic tensor to cover the contributions of all final-state particles. In this sense, calculating the hadronic tensor is so far the only way we know that Lattice QCD can serve the neutrino experiments.

The Euclidean hadronic tensor was formulated in the path-integral formalism to identify the origin of the Gottfried sum rule violation [4]. It is a current-current correlator in the nucleon with three-momentum transfer.

One can solve an inverse problem and find a solution from the Laplace transform [5]

$$\widetilde{W}_{\mu\nu}(\vec{q},\vec{p},\tau) = \int d\nu\, e^{-\nu\tau} W_{\mu\nu}(\vec{q},\vec{p},\nu). \tag{2}$$

This has been studied [6, 7] with the inverse algorithms such as the Backus-Gilbert, Maximum Entropy, and Bayesian Reconstruction methods.

For the hadronic tensor calculation, one important point is that it works in all the energy ranges (from elastic scattering to inelastic scattering and on to deep inelastic scattering). To check the validity of the hadronic tensor calculation, we studied the elastic scattering case [8]. For the elastic scattering case, the hadronic tensor $W_{\mu\nu}$ as a function of Q^2 is the product of the relevant nucleon form factors.

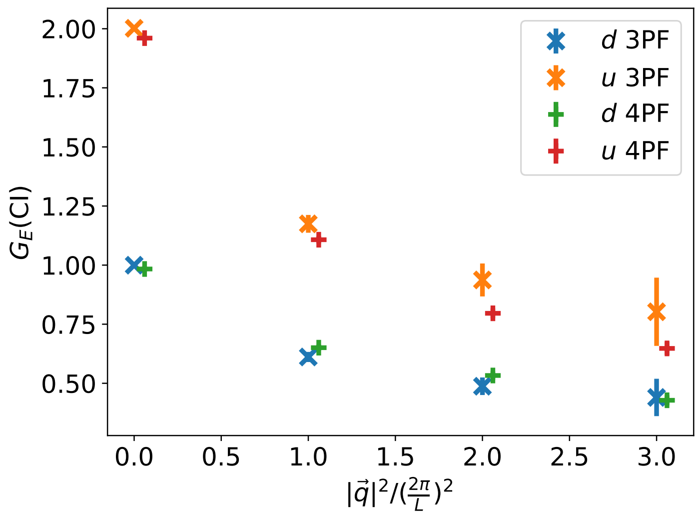

Figure 1: The comparison of the electric form factors (CI contributions only) calculated by using three-point functions and four-point functions for the first four momentum transfers (including zero) and for both u and d quarks.

Fig. 1 [8] shows that the electric form factors (connected insertions only) calculated by means of the three-point functions for both u and d quarks are found to be consistent within errors with those deduced from the hadronic tensor with the same vector charge current. The fact that they agree well in the $|\vec{q}|^2$ range up to $\sim$ 1GeV^2 is encouraging for future calculations of the neutrino-nucleon scattering in the resonance and inelastic regions. Reliable prediction of the cross sections is useful for the experiments to gauge the neutrino flux.

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