Tensor Network methods for lattice field theories

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1 Introduction

Among the most challenging computational problems today is that of performing first-principle non-perturbative simulations of strongly interacting gauge theories, such as QCD. While modern calculations of lattice QCD are able to reach sub-percent accuracy on many observables relevant to the HEP experimental program, there are many areas not accessible to current simulations including real-time dynamics, nonzero quark density, a topological θ-term, and gauge theories with chiral couplings. All of these involve sign-problems which make conventional Monte Carlo sampling inefficient. Tackling these problems will likely require finding completely new ways to simulate these theories compared to modern lattice field theory techniques.

Even for cases where standard lattice field theory simulations on classical computers are free of sign-problems, the Monte Carlo sampling of the partition function can become inefficient due to large potential barriers between sectors of distinct gauge topology. While much work is being done to alleviate this in preparation for upcoming exascale computing resources, a complete solution may not be found. This is another area where new simulation methods may have an advantage.

While quantum computing may be a promising long-term approach to these types of problems, some of the ideas related to QIS work may have a more timely impact on these areas. In particular, Tensor Network (TN) methods have had a large impact on low-dimensional models and are consistently being improved to the point that 4D models are starting to come within reach.

2 Tensor Network Methods

Perhaps the most well known TN method is the Density Matrix Renormalization Group (DMRG) method [1]. It has become the standard method for numerical computation of one dimensional systems in many contexts, beating out Monte Carlo methods. It was later discovered to be related to the Matrix Product States (MPS) [2–4] representation of a many-body wavefunction. MPS has been successfully applied to U(1) and SU(2) gauge models in one spatial dimension [5, 6]. In higher dimensions the analog is the Projected Entanglement Pair States (PEPS) [7] which, though computationally challenging, can provide a robust description of the ground state.

Tensor network methods are also being used to classically evaluate quantum circuits and are important for the near-term development and benchmarking of quantum algorithms.

One important advance was the development of a TN representation of abelian gauge theories with fermions in 2D. The Grassmann TRG [8] keeps track of fermion integrations and local signs very effectively. In spite of this progress, efficient TN computation in the presence of non-zero chemical potential and a topological θ term is still missing [9]. The definition of the tensor encodes all possible windings, however standard TRG iterations will not know which ones are the most important to keep. A blocking scheme which enforces the desired boundary conditions at each iteration and retains only the necessary windings is required. The ultimate goal is to find efficient TN representations and computation methods for non-abelian gauge theories with fermions. This will be the first step towards TN formulation of QCD in 4D.

Recent progress has seen methods developed for 4D applications applied to a scalar field theory [10, 11]. These methods are based on an anisotropic renormalization of tensors. These developments are bringing TN methods closer to the ultimate goal of simulating 4D gauge theories, however

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continued work in finding more efficient and stable algorithms is needed to achieve the goal of studying these theories with and without the addition of fermions. In addition to the computational methods required to construct efficient approximations of the full tensor network and scalable HPC code that can perform the contractions, advances may come from developing new formulations of field theories that are better suited for tensor networks.

3 New representations of field theories

Another promising byproduct of HEP research related to QIS is finding new formulations of field theories which could lead to a deeper understanding of their behaviors in addition to better classical simulation methods. Tensor networks offer one method to explore different formulations of field theories. Research in tensor networks has included exact analytical transformations from either Hamiltonian or Lagrangian formulations of field theories to their corresponding tensor representations. The relationship between tensor networks and the renormalization group method offers an alternative avenue to study the evolution and symmetries of the system directly from the TN representation. This allows one to explore the formulation and properties of theories at different scales in an alternate way to standard RG methods. Tensor network methods are thus not just a computational method, but an important tool for theoretical investigations of field theories.

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