Tensor Networks in High Energy Physics

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Since its precise predictions of the short distance physics in its early times, quantum field theory (QFT) has a successful track record and achieves its peak by establishing the standard model of particle physics. These successes have been achieved with numerous ideas from gauge theories to renormalization group (RG), which provided a theoretical framework for asymptotic freedom and quark confinement to coexist in a consistent way. At the time of these developments, lattice gauge theory was proposed as a practical way to do reliable QCD calculations. The lattice is a UV regulator and one needs to take the continuum limit (arbitrarily small bare coupling). This can be accomplished with importance sampling and Monte Carlo methods. Despite the success of these methods for calculating static properties of hadrons (e.g., form factors), they are problematic for theories with slowly running coupling constants which require very large lattices. RG methods have the potential capability to access smaller lattice spacings in larger physical volumes, which is important for understanding any effective QFT in general. Additionally, importance sampling falls short when one aims to study the dynamical properties, for example, the ab-initio description of fragmentation processes in hadron collisions. Replacing jet algorithms such as Pythia or Herwig by lattice calculations is a long term objective that would have a significant impact on collider data analysis. Applying a QCD Hamiltonian on a Hilbert space for quarks and gluons greatly exceeds what can be accomplished with high performance computers. New approaches are direly needed to overcome these obstacles.

QFTs have also been essential for quantum gravity (QG). Since the proposal of AdS/CFT correspondence by Maldacena, certain QFTs are believed to be dual to quantized gravity theories in one (or more) higher dimensions. Despite the tremendous progress, new perspectives are needed for a more systematic and detailed understanding. Indeed, recent studies suggest that these duality maps are tightly connected to concepts in quantum information, such as quantum error correction and quantum complexity. However, these recent connections are only the beginning of a long story, and there is a long path to make these precise and useful for a better understanding of how QG may work. For this, it is crucial to have a detailed understanding of the static and dynamical properties of QFTs, as these are one side of the duality (in fact, most of the time this is the part that we understand best).

All in all, the main problem is simulating QFTs (this encapsulates spin systems and even zero dimensional QFTs, i.e., quantum mechanics). Quantum information tools under development since the 90s provide partial answers, and promising future for a full understanding to crack this problem. In this LOI, we give a guide to help realize these goals with classical and quantum methods utilizing tensor networks (TNs). Tensor networks go back to K. Wilson's famous numerical RG solution to Kondo problem, which was a single impurity problem. In the 1990s, S. White noticed that these methods fell short due to neglecting the entanglement between coarse-grained blocks, and invented the density matrix renormalization group (DMRG). DMRG computes ground states of lattice Hamiltonians in one spatial dimension and is based on the matrix product state (MPS) TN. In the early 2000s, G. Vidal explained how to use

MPS to simulate dynamics of quantum systems under the constraint of low entanglement, while the MPS was generalized to projected entangled pair states (PEPS) for higher dimensional systems. G. Vidal also introduced a new type of TN called multi-scale entanglement renormalization ansatz (MERA) by introducing a renormalization scheme based on keeping track of local entanglement. Since then, TNs have gradually become one of the most effective numerical tools as ansatz states, and a conceptual tool to understand problems, even in areas outside of condensed matter physics, thanks to its foundations being built upon entanglement.

One major application of TNs for HEP is lattice gauge theory. An important step is discretizing and truncating the local quantum fields. Field truncations have been studied in the past years however much has to be done to reach the optimal values, potentially considering physical properties such as being in the low energy. Operating with optimal values is important because the complexity of TN algorithms grows sharply with the dimension of the local Hilbert spaces. In fact, classical TN methods have already been used to reformulate models studied by lattice gauge theorists. For compact matter fields and gauge groups this reformulation is automatically discrete and suitable to set up quantum computations or simulations. Symmetries and universality classes determine tensor selection rules which are preserved by truncations, and noise-robust implementation of Gauss' law are possible in any dimensions. Recent progress has been made regarding efficient implementations of the method in 2+1 and 3+1 dimensions (ATRG). A Lagrangian theory on a Euclidean space-time lattice can be smoothly transformed into a Hamiltonian theory with tensor deformations, which enables a good starting point for initiating quantum simulations, e.g, a real-time evolution calculation. Another way of simulating QFTs is using continuous tensor networks, which has been introduced by Verstraete and Cirac, for this particular reason. In fact, all these developments suggest a bigger goal to be pursued: a hybrid classical/quantum simulation method, in which initial states are prepared by variational TN algorithms, and time-evolution and measurements are performed by a combination of TN and quantum computing methods. The reward of this program is high, and it possesses theoretical challenges, such as how to contract tensor networks efficiently, how to compile them into a (noisy) quantum computer, how to perform efficient and reliable quantum state tomography utilizing TNs. Symmetry properties, low-energy features and finding effective Hamiltonians obeying these will not only improve the TN methods but also help for a more efficient quantum simulation. Furthermore, TNs can directly benefit lattice QFT calculations by assisting Monte Carlo simulations. Overall, we envision a roadmap of simulating QFTs with/without gauge invariance and seeking new approaches for physical problems such as confinement, etc., by starting from 1+1D models towards 3+1D, e.g., Schwinger model $\to$ 2+1D QCD $\to$ 3+1D QCD $\to$ QCD with additional matter content.

The other major application of TNs is in quantum gravity. A breakthrough has come in 2012 by B. Swingle, a connection has been established between AdS and MERA. Since then, this area has expanded including quantum error correction, cMERA, and quantum chaos. In all these, TNs have been used either directly or indirectly as an analytical/conceptual or numerical tool. The challenges for developing these early works include the same challenges as before, new ideas for tensor contractions and compilations are needed and using known/expected physical properties of QG is necessary. Furthermore, it is crucial to develop a systematic understanding of low/mid-energy excitations and thermal states with tensor networks, especially for MERA and cMERA. This is going to help with the long-standing problem of understanding the dynamics in AdS/MERA correspondence, which would lead to a general bulk-boundary map in the language of TNs. Furthermore, generalizing the correspondence to a full QG/QFT may require an out-of-box research on connections between tensor networks, quantum error correction and quantum simulation. A good starting point there is matrix models (0+1D matrix QFT, i.e., quantum mechanics with matrix degrees of freedom), e.g., a special instance of these models is shown to be dual to 11-dimensional M-theory, suggesting a more general correspondence to quantum gravity theories in spacetimes beyond AdS.