Lattice-QCD Determinations of Quark Masses and the Strong Coupling α_s

Fermilab Lattice, MILC, and TUMQCD Collaborations

September 1, 2020

EF Topical Groups: (check all that apply □/■)

■ (EF01) EW Physics: Higgs Boson Properties and Couplings
□ (EF02) EW Physics: Higgs Boson as a Portal to New Physics
■ (EF03) EW Physics: Heavy Flavor and Top-quark Physics
□ (EF04) EW Physics: Electroweak Precision Physics and Constraining New Physics
■ (EF05) QCD and Strong Interactions: Precision QCD
□ (EF06) QCD and Strong Interactions: Hadronic Structure and Forward QCD
□ (EF07) QCD and Strong Interactions: Heavy Ions
□ (EF08) BSM: Model-specific Explorations
□ (EF09) BSM: More General Explorations
□ (EF10) BSM: Dark Matter at Colliders

Other Topical Groups:

■ (RF01) Weak Decays of b and c Quarks

■ (TF02) Effective Field Theory Techniques
■ (TF05) Lattice Gauge Theory

■ (CompF2) Theoretical Calculations and Simulation

Contact Information:
Andreas S. Kronfeld (Theoretical Physics Department, Fermilab) [email]: ask@fnal.gov
on behalf of the Fermilab Lattice, MILC, and TUMQCD Collaborations

Fermilab Lattice, MILC, and TUMQCD Collaborations:
A. Bazavov, C. Bernard, N. Brambilla, C. DeTar, A.X. El-Khadra, E. Gámiz, Steven Gottlieb, U.M. Heller, W.I. Jay, J. Komijani, A.S. Kronfeld, J. Laiho, P.B. Mackenzie, E.T. Neil, P. Petreczky, J.N. Simone, R.L. Sugar, D. Toussaint, A. Vairo, A. Vaquero Avilés-Casco, J.H. Weber, R.S. Van de Water

Lattice-QCD Determinations of Quark Masses and the Strong Coupling α_s

Quantum chromdynamics (QCD) as a stand-alone theory has $1 + n_f + 1$ free parameters that must be set from experimental measurements, complemented with theoretical calculations connecting the measurements to the QCD Lagrangian. These parameters are the strong coupling α_s (at some scale), the n_f quark masses (six so far), and the vacuum angle θ that multiplies the gauge-invariant CP-violating combination of gluon fields (in the basis with real quark masses). The importance of these parameters is pervasive in particle and nuclear physics. For example, for the 2013 Snowmass Study, Lepage, Mackenzie, and Peskin [1] studied how precise α_s and the bottom and charmed quark masses should be to serve the needs of Higgs-boson measurements at the ILC or other lepton-collider Higgs factory. In this LOI, we assess what lattice QCD can contribute to the determination of α_s and quark masses.

In lattice QCD, the determination of quark masses is conceptually very simple: compute hadron masses and adjust the bare quark masses so that n_f hadron masses agree with experiment. Here, $n_f = 5$, because the top quark mass is much too large compared with the ultraviolet cutoff associated with the nonzero lattice spacing. Ideally, the chosen hadron masses are sensitive to one quark mass but not so much to the others. Examples include the pion and kaon masses for light and strange quarks and either quarkonium or heavy-light meson masses for b and c quarks. In addition to these n_f masses, a further hadronic observable is needed to convert from lattice units to physical units. Ideally, this mass is not very sensitive to quark masses or can be combined with the pion or kaon mass to set two parameters by interpolating in a plane.

This procedure is complicated by mass renormalization in quantum field theory. It doesn't make much sense to quote bare lattice masses, and perturbative-QCD calculations are usually renormalized in the $\overline{\text{MS}}$ scheme of dimensional regularization. Here we focus on three approaches:

1. renormalize the bare mass in a regulator-independent scheme and, after taking the continuum limit, convert to $\overline{\text{MS}}$ at a high scale;

2. compute quarkonium observables, take the continuum limit, and analyze the results with $\overline{\text{MS}}$ perturbation theory;

3. exploit effective field theory to bridge from lattice QCD to continuum QCD—in practice "continuum" means $\overline{\text{MS}}$.

The last two methods are useful only for heavy quarks, employing perturbation theory the scale m_Q. Heavy-quark masses can be propagated, however, to light-quark masses using regulator- and renormalization-independent ratios of quark masses. These lattice-QCD methods do not require lattice perturbation theory (which is cumbersome and, thus, available at one or at most two loops). Of course, lattice perturbation theory can be used in the analysis to reduce systematic uncertainties.

Let us compare three results in the $\overline{\text{MS}}$ scheme, one from each method, evaluated at the scale in parentheses::

$$m_c(3\text{ GeV}) = 989.6(6.1)\text{ MeV [2]}, \tag{1}$$

$$m_c(3\text{ GeV}) = 985.1(6.3)\text{ MeV [3]}, \tag{2}$$

$$m_c(3\text{ GeV}) = 983.7(5.6)\text{ MeV [4]}, \tag{3}$$

The dominant uncertainties differ greatly. For example, the uncertainty from truncating of perturbation theory is important in the second determination [3], but subdominant in the first [2] and negligible in the last [4]. From recent reviews [5, 6], one can see that other results for the second method [7–9] agree well with the displayed result. The uncertainties on the other quark masses range from subpercent (bottom) to

2% (up). They still fall short of the ILC target [1] but seem sufficiently precise for LHC measurements of Higgs-boson branching fractions.

It is worth noting that the most precise results [2–4] all depend on the same set of ensembles of gauge-field configurations, generated by the MILC Collaboration [10, 11] and often known as the "HISQ ensembles". These ensembles have the widest available range of lattice spacing, uniformly high statistics, and physical light-quark masses in the sea. It will be important for the community to support other lattice-QCD collaborations in their efforts to generate a set of ensembles of similar power. Other precise work [9] also employs the HISQ approach.

Most determinations of the strong coupling α_s follow strategies similar to those for quark masses. The observables are now dimensionless and contain a short distance, so they admit a perturbative expansion, much like the observables used to determine α_s from high-energy scattering and heavy-particle decays. The most obvious short distance in lattice QCD is the lattice spacing. Thus, in addition to methods based on the continuum limit, it is possible to apply perturbation theory to small Wilson loops and study various combinations with differing ultraviolet sensitivity [7, 12, 13]. On the other hand, a continuum-limit observable must contain a short-distance scale. In this way, the quarkonium observables that yield m_b and m_c also yield α_s. Another approach is to use a small finite volume, L^3, and study the running of $\alpha_s(1/L)$ over a wide range of scales $(1/L)$ [14, 15].

Considering the wide variety of methods, the reviews quote lattice-QCD averages

$$\alpha_s(m_Z) = 0.1182 \pm 0.0008 \text{ [5]}, \tag{4}$$

$$\alpha_s(m_Z) = 0.11803\,^{+\,0.00047}_{-\,0.00068} \text{ [6]}, \tag{5}$$

for 5 active flavors. Note that Ref. [5] finds a combined total uncertainty 0.0006 from the lattice-QCD results alone, in agreement with Ref. [6]. The error bar quoted in Eq. (4) comes from an assessment of the common uncertainty in all (lattice or non-lattice) α_s determinations, namely the truncation of perturbation theory.

During the Snowmass Study, we LOI authors are interested in exploring what the lattice-QCD precision of the quark masses and α_s implies for QCD at high-energy colliders. Note that some collider determinations of α_s agree well with Eqs. (4) and (5), others do not. One should bear in mind that the perturbation theory entering most lattice-QCD determinations is relatively simple: the pattern of nonperturbative contributions is understood and can be controlled. Resummation of large logarithms seems to be unnecessary, as a rule, although at higher orders one should be attentive to to soft scales such as $\alpha_s m_Q$ or α_s/L. In several meetings of EF05 and EF06, participants have eagerly anticipated future lattice-QCD calculations of parton distribution functions (PDFs). We share this enthusiasm. To further the understanding of these yet-to-come calculations, we point out that the uncertainty budgets for α_s, m_b, and m_c share many features with the PDFs but are *much simpler*. Thus, the already-published results for these quantities provide non-experts a place to deepen their practical understanding of lattice QCD.

In our own research, we are pursuing two new determinations of α_s. One (by Fermilab Lattice and MILC) is an independent analysis of quarkonium moments, with data already generated. The other (by TUMQCD) is an analysis of the static energy [16] exploiting some of the ideas from our work on quark masses [4], again with data already generated. Both analyses are of high priority. On the other hand, the quark masses quoted [Eqs. (1)–(3)] are precise enough to last until the ILC (or other lepton-collider Higgs factory). At that time, the total uncertainties in α_s, m_b, an m_c should be (approximately) halved [1]. To reach such precision, interesting theoretical issues related to electromagnetic effects must be addressed. In the meantime, it is important for other lattice-QCD collaborations to match our precision on quark masses, and for everyone to improve the uncertainties on α_s, both quantitatively and qualitatively.

References

[1] G. P. Lepage, P. B. Mackenzie, and M. E. Peskin, (2014), arXiv:1404.0319 [hep-ph] .

[2] A. T. Lytle, C. T. H. Davies, D. Hatton, G. P. Lepage, and C. Sturm (HPQCD), Phys. Rev. **D98**, 014513 (2018), arXiv:1805.06225 [hep-lat] .

[3] B. Chakraborty, C. T. H. Davies, B. Galloway, P. Knecht, J. Koponen, G. C. Donald, R. J. Dowdall, G. P. Lepage, and C. McNeile, Phys. Rev. **D91**, 054508 (2015), arXiv:1408.4169 [hep-lat] .

[4] A. Bazavov *et al.* (Fermilab Lattice, MILC, TUMQCD), Phys. Rev. **D98**, 054517 (2018), arXiv:1802.04248 [hep-lat] .

[5] S. Aoki *et al.* (Flavour Lattice Averaging Group (FLAG)), Eur. Phys. J. C **80**, 113 (2020), arXiv:1902.08191 [hep-lat] .

[6] J. Komijani, P. Petreczky, and J. H. Weber, Prog. Part. Nucl. Phys. **113**, 103788 (2020), arXiv:2003.11703 [hep-lat] .

[7] C. McNeile, C. T. H. Davies, E. Follana, K. Hornbostel, and G. P. Lepage, Phys. Rev. **D82**, 034512 (2010), arXiv:1004.4285 [hep-lat] .

[8] K. Nakayama, B. Fahy, and S. Hashimoto, Phys. Rev. **D94**, 054507 (2016), arXiv:1606.01002 [hep-lat] .

[9] P. Petreczky and J. Weber, Phys. Rev. D **100**, 034519 (2019), arXiv:1901.06424 [hep-lat] .

[10] A. Bazavov *et al.* (MILC), Phys. Rev. D **87**, 054505 (2013), arXiv:1212.4768 [hep-lat] .

[11] A. Bazavov *et al.* (Fermilab Lattice, MILC), Phys. Rev. D **98**, 074512 (2018), arXiv:1712.09262 [hep-lat] .

[12] C. Davies, K. Hornbostel, I. Kendall, G. Lepage, C. McNeile, J. Shigemitsu, and H. Trottier (HPQCD), Phys. Rev. D **78**, 114507 (2008), arXiv:0807.1687 [hep-lat] .

[13] K. Maltman, D. Leinweber, P. Moran, and A. Sternbeck, Phys. Rev. **D78**, 114504 (2008), arXiv:0807.2020 [hep-lat] .

[14] S. Aoki *et al.* (PACS-CS), JHEP **10**, 053 (2009), arXiv:0906.3906 [hep-lat] .

[15] M. Bruno, M. Dalla Brida, P. Fritzsch, T. Korzec, A. Ramos, S. Schaefer, H. Simma, S. Sint, and R. Sommer (ALPHA), Phys. Rev. Lett. **119**, 102001 (2017), arXiv:1706.03821 [hep-lat] .

[16] A. Bazavov, N. Brambilla, X. Garcia i Tormo, P. Petreczky, J. Soto, A. Vairo, and J. H. Weber (TUMQCD), Phys. Rev. D **100**, 114511 (2019), arXiv:1907.11747 [hep-lat] .