Gluon Parton Distribution Functions from Lattice QCD

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Gluon parton distribution functions (PDFs) characterize the momentum distribution of gluons inside the hadrons, and are important inputs for making theory predictions of gluon-initiated processes at hadron colliders. Our present knowledge of gluon PDFs predominantly comes from the global analysis of experimental data ranging from charm production in deep-inelastic scattering to top quark pair production at the LHC. Despite the efforts being made, the results of different groups exhibit important discrepancies that need to be resolved. For example, in the large-x region the difference in the unpolarized gluon PDF from NNPDF3.1 and CT14 can reach $\sim 2\sigma$ level [1]. This might be improved when more experimental data are collected at ongoing and future experimental facilities such as the Electron-Ion Collider, a particular focus of which is to study the gluonic structure of hadrons.

On the other hand, there have also been efforts to understanding the gluonic structure of hadrons from lattice QCD. Such efforts are mostly focused on the calculation of leading moments of the gluon PDFs, e.g., the total momentum and spin of the hadron carried by gluons [2–5].

Here we propose to perform a systematic study of the x-dependence of gluon PDFs from lattice QCD, based on the methodology developed recently (for a review see, e.g., [6, 7]) that allows to extract parton properties of hadrons from Euclidean correlation functions. The starting point is the equal-time correlation function $\langle PS|O_g(z,0)|PS\rangle$, where $O_g(z,0)$ is a spatial nonlocal operator constructed from gluon fields, and becomes the standard gluon PDF operator under an infinite Lorentz boost. The Fourier transform of the above correlation function defines a finite momentum distribution, which can be converted to the standard gluon PDF via a factorization formula accurate up to power corrections that are suppressed by the momentum of the hadron.

Following the above methodology, some exploratory studies on gluon PDFs have been performed [8, 9]. However, several important issues remain to be settled and will constitute the main body of the present study. To be concrete, we will investigate the following:

1. The most appropriate operator choice for the calculation. Due to the complication of simulating gluon observables on the lattice, a well-suited choice of $O_g(z,0)$ will greatly facilitate the calculation. This can already be seen from the exploratory studies in [8, 9], where in the latter an improved gluon operator that is multiplicatively renormalized is used, resulting in a better signal-to-noise ratio at the same hadron momenta even though the number of measurements is smaller than that in the former. In Refs. [10, 11], several operators with a simple renormalization under lattice regularization have been proposed to study the gluon PDFs. However, exploratory lattice studies

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have shown that the signals for such operators are actually not as good as expected. This could be due to potential contaminations from higher-twist operators, as also indicated by perturbative analysis [12]. We will reinvestigate the proposed operators, and figure out appropriate combinations that both have simple renormalization and do not suffer from higher-twist contaminations.

2. Renormalization strategy. The renormalization of gluon operators on the lattice in general is an unsettled issue. The main reason is that the widely used lattice renormalization scheme requires calculating the operator matrix elements in an off-shell external state, which brings in severe problems of gauge invariance when the external state is a gauge particle such as the gluon. Indeed, in Ref. [11] it has been observed that for the correlation function $\langle PS|O_g(z,0)|PS\rangle$ gauge-variant operators play a vital role in properly defining the lattice renormalization scheme and the corresponding matching to the continuum. However, this can be avoided if we take full advantage of the fact that the proposed operators are multiplicatively renormalizable, and thus all ultraviolet divergences can be removed by dividing an on-shell matrix element of the same operator. Of course, by doing so we have to keep in mind that the renormalization shall not introduce any extra nonperturbative (infrared) effects at large-z. In Ref. [13], a renormalization scheme that fulfills the above requirements has been proposed for lattice studies of quark PDFs. In this study, we will extend that proposal to the gluon case.

3. Control of lattice systematics. Calculating gluon observables on the lattice is more difficult than calculating the quark ones, as the former have worse signal-to-noise ratios, and thus require very high statistics to reveal a signal. In the existing exploratory studies [8, 9], the calculation is only done on one ensemble with a single lattice spacing and two valence quark masses or pion masses. There are a number of systematics yet to be studied. We plan to extend the analysis to ensembles with multiple lattice spacings and pion masses, and study the relevant systematics.

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