Precision Moments of Strange Parton Distribution Functions from Lattice QCD

Tanmoy Bhattacharya,[1] Rajan Gupta,[1] Huey-Wen Lin,[2,3] Santanu Mondal,[1] Boram Yoon,[1] and Rui Zhang[2,3]
[1]*Los Alamos National Laboratory, Theoretical Division T-2, Los Alamos, NM 87545, USA*
[2]*Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824*
[3]*Department of Computational Mathematics, Science and Engineering,*
Michigan State University, East Lansing, MI 48824, USA

Motivation: PDFs provide a universal description of hadronic constituents as well as critical inputs for the discovery of the Higgs boson found at the Large Hadron Collider (LHC) through proton-proton collisions [1, 2]. Despite this great victory, the LHC has many tasks remaining, and the focus of the future Runs 3–5 will be to search for physics beyond the Standard Model. In order to probe new physics at the LHC, we need to improve the precision with which we know the Standard-Model background, so as to discriminate between these and new-physics signatures. Unfortunately, our knowledge of many Higgs-production cross sections remains dominated by PDF uncertainties. Improvement of current PDF uncertainties is important to assist LHC new-physics searches.

In addition to their applications to the energy frontier, PDFs also reveal nontrivial structure inside the nucleon, such as the momentum and spin distributions of partons. Many ongoing and planned experiments at facilities around the world, such as Brookhaven and Jefferson Laboratory in the United States, GSI in Germany, J-PARC in Japan, or a future electric-ion collider (EIC), are set to explore the less-known kinematics of the nucleon structure and more.

In order to distinguish the flavor content of the PDFs, one would need to use nuclear data, such as neutrino scattering off heavy nuclei, and the current understanding of medium corrections in these cases is limited. Thus, the uncertainty in the strange distribution in PDFs remains large. In some cases, the assumption $\overline{s}(x) = s(x)$ made in global analyses can agree with the data due to the large uncertainty. At the LHC, strangeness can be extracted through the $W + c$ associated-production channel, but their results are rather puzzling. For example, ATLAS gets $(s + \overline{s})/(2\overline{d}) = 0.96^{+0.26}_{-0.30}$ at $Q^2 = 1.9$ GeV^2 and $x = 0.23$ [3]. CMS performs a global analysis with deep-inelastic scattering (DIS) data and the muon-charge asymmetry in W production at the LHC to extract the ratios of the total integral of strange and anti-strange to the sum of the anti-up and -down, at $Q^2 = 20$ GeV^2 finding it to be $0.52^{+0.18}_{-0.15}$ [4]. Future high-luminosity studies may help to improve our knowledge of the strangeness. In the polarized case, SU(3)-flavor symmetry is often assumed due to lack of precision experimental data. We learn from the unpolarized case that this assumption introduces an underestimated uncertainty. In addition, there has been long debate concerning how big the intrinsic charm contribution is or whether other heavy flavors contribute. Again, the data is too inconclusive to discriminate between various proposed QCD models.

However, due to the difficulty of working in the strong-interaction regime, after decades of experiments and theoretical efforts, there still remain many unknowns. Experiments, such as deep inelastic scattering of electrons off nucleons generate cross sections containing information about nucleon structure, but they are not complete. Although there exist a variety of model approaches to treat the structure functions, a nonperturbative approach from first principles, such as lattice QCD (LQCD), provides hope to resolve many of the outstanding theoretical disagreements and provide information in regions that are unknown or difficult to observe in experiments.

Lattice QCD: A nonperturbative approach using first principles, such as LQCD, provides hope to resolve many of the outstanding theoretical disagreements and provide information in regions that are unknown or difficult to observe in experiments. LQCD is a regularization of continuum QCD using a discretized four-dimensional spacetime; it contains a small number of natural parameters, the strong coupling constant and quark masses. Unlike continuum QCD, LQCD works in Euclidean spacetime (rather than Minkowski), and the coupling and quark masses can be set differently than those in our universe. The theory contains two scales that are absent in continuum QCD, one ultraviolet (the lattice spacing a) and one infrared (the spatial extent of the box L); this setup keeps the number of degrees of freedom finite so that LQCD can be solved on a computer. For observables that have a well-defined operator in the Euclidean path integral for numerical integration, we can find their values in continuum QCD by taking the limits $a \to 0$, $L \to \infty$ and $m_q \to m_q^{\rm phys}$. LQCD is a natural tool to study the form factors needed for the neutrino experiments, since quarks and gluons are the fundamental degrees of freedom.

There has been significant progress in LQCD in the past decade. It is now fairly common to have at least one calculation using the physical pion mass with realistic light-quark loops in the QCD vacuum. This overcomes the long-standing obstacle of approximating the real world with unphysically heavy quark masses due to the high computational cost of light quark masses. As a result, more precise QCD quantities can be directly calculated on the lattice. For example, flavor physics from lattice QCD has been playing an important role in determining Standard-Model inputs in the heavy-meson sector: B- and D-meson decay constants, B-meson mixing parameters and more. For lighter mesons, such as pion and kaon systems, lattice QCD can determine many related quantities pretty reliably. In the PDG, the most precise determination of quark masses and the strong-coupling constant are from LQCD calculations.

However, nucleon matrix-element calculations on the lattice are harder in general because of the following problems. Firstly, we face a signal-to-noise problem. In Euclidean space the nucleon signal-to-noise ratio scales like $e^{(-M_N+3M_\pi/2)}$ with Euclidean time; we need to extract the ground-state signal before the noise becomes overwhelming. The first excited state $N(1440)$ is nearby; therefore, we have to be careful to distinguish the ground and excited signals to avoid contamination and wait for larger time where the excited-state signal is relatively diminished, if necessary. However, this is at odds with the first problem; as we wait for larger time to avoid excited states, the signal-to-noise ratio decreases. Thus, we need high statistics and longer trajectories to have reliable numbers for experiments. Second, chiral perturbation theory (XPT) is more difficult; for example, there are multiple expansions for XPT, and some of them may have poor convergence, bringing the validity of chiral forms into question at the heaviest relevant pion mass. Thankfully, this issue may soon become moot in the era of physical pion mass. Also, larger volumes are required (in comparison to the meson case) to avoid systematic uncertainties due to the finite volume.

Despite all these challenges in calculating nucleon-related quantities, some groups are making progress pushing the limits of precision nucleon matrix elements for upcoming experiments, even though these require an order of magnitude more computational resources to get to few-percent errors relative to the meson case. For example, Precision Neutron-Decay Matrix Elements (PNDME), a collaboration that majority of this project team founded in 2011, has been at the frontier pushing the standard established by the Flavor Lattice Averaging Group (FLAG) [5–7], whose numbers (such as decay constants and quark masses) have been widely taken as SM inputs to probe new physics and other applications. PNDME's nucleon-charge calculations, published in Ref. [8, 9], are the only ones that have all green-star ratings according to the FLAG standard [5–7], indicating to those who may not be lattice experts that the calculation has the highest degree of systematic control. We also implemented an additional systematic due to excited-state contamination which is significant in nucleon matrix elements. With these precision nucleon inputs, we are able to combine the lattice inputs with current electric dipole moments (a CP-violation test in electron and neutron systems) and set an upper bound for a certain new-physics model (split SUSY). We are also using our lattice inputs (isovector tensor and scalar charges), along with past neutron beta-decay experiments, to set lower bounds on the masses of potential TeV-scale particles through effective field theory for the LHC and future colliders to find. In this project, we will adapt these lattice-QCD techniques to provide the precision nucleon strange moments needed to improve the strange-quark PDFs.

Proposed Work and Anticipated Impacts Although the moment approach is not new, there are a very limited number of good calculations suitable to be used in PDF analysis, even just considering isovector channels. There are only a few first strange moments done with dynamical fermions in the past decade, and none have full systematic control. There is significant room for improvement and for the addition of the strange contributions. Furthermore, a strange second-moment calculation from lattice QCD has not been done yet. This effort will be in parallel with the recent study of the Bjorken-x dependence of the PDFs, which will have an immediate impact in the large-x region, while the moments can allow us to constrain the small-x region better than the current techniques. We plan to carry out multiple first and second moments calculations using multiple lattice spacing, $a \approx 0.15, 0.12, 0.09$, and 0.06 fm with pion mass ranging from 310 to 135 MeV. Our upcoming calculation will address all the lattice systematics which is a big step forward to full systematic control.

The anticipated impact has been studied in a joint-community white-paper exercise [10] between people in lattice QCD and global PDF analysis. An initial estimate of the potential impact of present and future lattice-QCD calculations on global unpolarized and polarized PDF fits using three possible error-budget scenarios (A: 70%, B: 50%, C: 20%) for the strange-quark unpolarized and polarized PDF moments computed with NNPDFpol1.1. Take polarized strange moment for example. For scenarios A and B, there is only a very moderate reduction (or even a slight increase) in the PDF uncertainties, seemingly at odds with the results for their moments. The reason is that the first PDF moments alone provide only limited information on the shapes of the PDFs, and therefore in some cases, one will find a large error reduction on the moments (since these are the fitted quantifies) and not on the PDFs themselves (which are only indirectly constrained). Once the lattice-QCD pseudo-data uncertainties decrease beyond a certain level, such as scenario C, the PDF uncertainties can decreases by up to a factor 3 for $\Delta s^+(x,Q)$.

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[1] S. Chatrchyan *et al.* (CMS), Science **338**, 1569 (2012).
[2] G. Aad *et al.* (ATLAS), Science **338**, 1576 (2012).
[3] G. Aad *et al.* (ATLAS), JHEP **05**, 068 (2014), arXiv:1402.6263 [hep-ex] .
[4] S. Chatrchyan *et al.* (CMS), Phys. Rev. **D90**, 032004 (2014), arXiv:1312.6283 [hep-ex] .
[5] S. Aoki *et al.*, Eur. Phys. J. **C77**, 112 (2017), arXiv:1607.00299 [hep-lat] .
[6] S. Aoki *et al.*, Eur. Phys. J. **C74**, 2890 (2014), arXiv:1310.8555 [hep-lat] .
[7] G. Colangelo *et al.*, Eur. Phys. J. **C71**, 1695 (2011), arXiv:1011.4408 [hep-lat] .

[8] T. Bhattacharya, V. Cirigliano, S. Cohen, R. Gupta, H.-W. Lin, and B. Yoon, Phys. Rev. **D94**, 054508 (2016), arXiv:1606.07049 [hep-lat] .

[9] T. Bhattacharya, V. Cirigliano, S. Cohen, R. Gupta, A. Joseph, H.-W. Lin, and B. Yoon (PNDME), Phys. Rev. **D92**, 094511 (2015), arXiv:1506.06411 [hep-lat] .

[10] H.-W. Lin *et al.*, Prog. Part. Nucl. Phys. **100**, 107 (2018), arXiv:1711.07916 [hep-ph] .