# Small-x parton physics on lattice

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In recent years, there has been a lot of interest in small-x physics [1–6], and calculating parton physics (ref.) on lattice [7–9].

At small-$x$, the gluon density grows due to the Bremsstrahlung radiation of a large number of low-momentum gluons inside the proton and heavy nuclei. These low-momentum gluons are referred as the small-$x$ gluons, with $x$ being the longitudinal momentum fraction w.r.t. the parent hadron. Nevertheless, the gluon density in the small-$x$ region can not grow arbitrarily large due to an intriguing mechanism in QCD. First, the typical transverse momentum of the gluons grows at small-$x$ so that the corresponding gluon size becomes smaller in the transverse space, and more gluons can fit into a fixed transverse area. Besides, the gluon recombination through the non-linear coupling will become important when its density becomes sufficiently high. When the Bremsstrahlung radiation and the recombination reach a detailed balance, the gluon number density is expected to saturate. The saturation of the gluon density might be universal in the sense that it is independent of the original states of the color sources. That is to say that the saturated gluon state only depends on some global properties of the system, but it is independent of the dynamical details. The study of the emergent properties of the dense gluon systems in protons and heavy nuclei has become the forefront of small-$x$ research in recent years.

Partons cannot usually be calculated on Euclidean lattice, due to that the usual parton distributions are light-front correlations in Minkowski space. Time-dependent correlations are usually difficult to simulate using Monte Carlo methods. However, large-momentum effective theory (LaMET) proposed by one of us uses a Euclidean formulation of partons in which the operators are time-independent and the external states have infinite momentum [7, 8]. By approximating the infinite-momentum by a finite one, and systematically expanding the result in terms of the inverse large momentum, one can formulate a parton calculation in lattice QCD. Much progress has been made recently in this direction.

However, just like the high-energy experiments, the LaMET calculation requires generating large-momentum states on lattice. Smaller the x that one intends to study, the larger the momentum of the hadron must be, and therefore, the smaller the lattice spacing a in the direction of motion. For practical calculations, there is a strong limit on how small x one can get with a given computer resources. In this proposal, we suggest tackling the small x problem on lattice in three different approaches

1. Explore special methods to create large-momentum hadrons on lattice. For example, one possibility is to use asymmetric lattice configruations in which the longitudinal direction has much smaller lattice spacing than the transverse direction.

2. Find an effective theory which can go beyond the present formulation of LaMET. In this approach, one tries to formulate a new approach in which one does not need very small lattice spacing but a new type of sources generated from renormalization group running of thinning the degrees of freedom in the longitudinal direction.

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3. Since the small-x saturation happens for nuclei as well, we propose to calculate the nuclear distribution on lattice using the standard LaMET method. And explore for how big a nucleus, one can make a sensible calculation.

We suggest to bring people from both the small-x and lattice QCD communities together, and discuss possible rigorous approaches to small-x physics using the fundamental QCD theory.

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