Letter of Interest for EF06: Parton distribution functions from lattice QCD

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1 Motivation

In the partonic picture of hadron, PDF can be viewed as the probability density $q(x)$ for finding a particle with a certain longitudinal momentum fraction x in a fast moving hadron. The PDFs provide useful information on the structure of the hadrons. In addition, accurate knowledge of PDFs is necessary to reduce the systematic uncertainties in the electroweak precision studies at the high-luminosity LHC. [1]

Quantum Chromodynamics (QCD) is the fundamental theory for strong interactions, and is responsible for holding the quarks and gluons together to form hadrons. Therefore, PDFs are completely determined by this theory. However, the quarks and the gluons interact strongly within hadrons. The structure of hadrons cannot be determined with perturbative method. Traditionally, PDFs are obtained from global analysis of high energy hard scattering experiments. Based on QCD factorization, many hard-scattering cross sections can be expressed in terms of the convolution of the PDFs and some short-distance, perturbative, matrix elements. Lattice QCD framework provides a numerical approach to calculate non-perturbative hadronic observables from the first principles. There are a lot of developments in methodology and many works have emerged in calculating PDFs using lattice QCD. Following our existing work [2, 3], we will continue to improve the lattice calculation for pions PDFs and extend to proton PDFs, aiming at improving the precision of PDFs used for LHC electroweak precision studies. We will also explore developments in the methodology of lattice calculation of PDFs.

2 Method

The method for calculating PDF on the lattice is not straight forward. Conventionally, PDF is defined as matrix elements of hadrons with non-local operators separated in the light-cone direction in the Minkowski space-time. Such matrix elements cannot be directly calculated in the Euclidean space-time lattice. The initial method to circumvent this difficulty is to design local operators, whose matrix elements give moments of the PDFs. The method works and one is able to calculate the first three moments of PDFs. [4, 5, 6] However, it is not very feasible to calculate higher moments of PDFs due to the operators for higher moments mix with lower dimensional operators, whose contributions are usually much larger than the original operator, and are very hard to subtract. One improvement is suggested in Ref. [7], it was shown that using non-local operators instead of local operators can ease the operator mixing issue. Another improvement is observed in Ref. [8]. It was proposed that moving hadron states can be used in the lattice calculation instead of stationary states. With the fast moving hadron states, quasi-PDFs are introduced using space direction non-local operators that are similar to the ones that were used to define PDFs in Minkowski space-time. This approach has the potential to obtain the Bjorken-x dependence of PDFs in additional to the first three moments. Along this direction, some additional developments were made. Ref. [9] recognized the similarity between the lattice calculation of quasi-PDFs and the cross sections measured in high energy experiments. Then different kinds of non-local operators were studied. Ref. [10] suggest that using the same non-local operator, but measured using hadron states with different momentum can also help extracting PDFs.

3 PDFs of pion and proton

In our previous works [2, 3], we have calculated the valence PDF of pion from lattice QCD, with heavier than physical pion mass $m_\pi = 300$ MeV. We are extending the calculation to lattices with physical pion mass, and also performing calculation for proton PDFs. There are several important aspect for the PDF calculation, which we are be actively working on:

- Lattice artifacts (non-zero lattice spacing effects): In our existing studies, we have used two lattices with very small lattice spacings $a = 0.06$ fm and $a = 0.04$ fm to control the lattice artifacts, continuum limit is studied. The lattice operator renormalization is also studied extensively.

- Excited state effects: Proton states and pion state with non-zero momentum suffers signal to noise ratio problem when the separation between the operator and the hadron source/sink increases. This is probably the most important . Multi-state fit is employed to control the excited state effects at modest separation. The energies of the first excited states from the the fit match the expectation well. Using multiple hadron sources can better control the excited state effects better.

- Higher twist effect: This effect is caused by the non-local operators used in lattice calculations are not in the light-cone direction. Therefore, the size of the operators needs to be constraint in the calculation. Larger hadron momentum will help extracting more information with limited operator size.

- Matching between PDFs and lattice quasi-PDFs: The matching relies on a perturbatively calculated kernel. Currently, one loop calculation is available. [11] Higher order calculation for some of the operators used in the lattice calculation may be available in the future.

- Sea quark and gluon PDFs: Calculation for this is still in its early stage. We plan to explore in these areas.

4 Meson distribution amplitudes

Meson distribution amplitudes (DAs) are similar to the PDFs. They also provide information for the structure of mesons and can serve as input for some high energy processes involve these mesons. In addition, this can be testing ground for new method of lattice calculation.

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