Snowmass Letter of Interest:
Leptonic Sum Rules

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Sum rules in the lepton sector provide an extremely valuable tool to classify flavour models in terms of relations between neutrino masses, mixing angles and CP-violating phases, which can be tested in experiment.

Frontiers: NF01, NF03, NF05, TF11

I. INTRODUCTION

The origin of the observed pattern of neutrino masses and mixing and the status of leptonic CP violation are among the "big" open questions in particle physics. Considerable efforts have been made in the past years towards answering these long-standing questions from symmetry principles. In particular, discrete non-Abelian family symmetries have been thoroughly investigated as they may be responsible for the observed flavour structure in the lepton sector (for reviews see [1–6]). A salient feature of the models based on such symmetries is a prediction of (i) specific values of the neutrino masses, mixing angles and CP-violating phases and/or (ii) certain relations between these observables. Here we focus on two different types of such correlations: (i) *neutrino mass sum rules* relate the three (complex) neutrino mass eigenvalues to each other and (ii) *leptonic mixing sum rules* connect the leptonic mixing angles as well as the CP-violating phases.

Studying these sum rules further in the recently proposed new class of models based on modular symmetries [7] as well as analysing their implications and testability at current and near-future experiments is essential to identify benchmarks to be tested. This can ultimately guide us to unveil the theory behind the neutrino sector and the origin of flavour.

II. MASS SUM RULES

Neutrino mass sum rules are relations between the three complex neutrino mass eigenvalues $\tilde{m}_i = m_i \exp(\mathrm{i}\alpha_i)$ (with Majorana phases α_1, α_2 and we choose α_3 to be unphysical), e.g., $\tilde{m}_1 + \tilde{m}_2 + \tilde{m}_3 = 0$ or $\tilde{m}_1^{-1} + \tilde{m}_2^{-1} + \tilde{m}_3^{-1} = 0$. Geometrically the sum rule can be interpreted as a triangle in the complex plane. Neutrino mass sum rules are present in over 60 flavour models in the literature motivating a detailed study of their predictions and properties to obtain more insights in the neutrino sector.

Since mass sum rules involve the Majorana phases, the obvious observable which is constrained is the effective Majorana mass $|m_{ee}|$ in neutrinoless double beta decay. However, also other observables like the absolute neutrino mass scale as measured in cosmology or in beta decay experiments are affected by the existence of a mass sum rule in terms of a lower bound. Some mass sum rules even predict a specific mass ordering, as the relation $\tilde{m}_1 + 2\,\tilde{m}_2 - \tilde{m}_3 = 0$ that can only be fulfilled in the case of normal mass ordering.

Mass sum rules have been studied in the past [8–13] but only in recent years more systematic categorisations and analyses of their predictions have been performed [14–16]. In fact, sum rules are usually subject to corrections due to higher-dimensional operators and renormalisation group running effects. In [17, 18] phenomenological studies of these corrections to mass sum rules have been conducted with the result that their predictions are rather stable under

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corrections. Finally, in [19] it was proven that the origin of mass sum rules is solely a parameter reduction, and they are not directly related to a residual symmetry as naively suspected. From the experimental side it has been shown in [20] that neutrino mass sum rules can be used to sensibly plan the experimental stages of neutrinoless double beta decay experiments, and how results from intermediate stages can be used to constrain whole groups of theoretical scenarios.

Neutrino mass sum rules have vast implications and allow to explore synergies between neutrino theory and experiment.

III. MIXING SUM RULES

Another class of sum rules are leptonic mixing sum rules. These are relations among the mixing angles and the CP-violating phases, for reviews including references to early works on the subject, see, for instance, [3–5] and for recent systematic studies, see [21–28]. An exhaustive literature review is beyond the scope of this LOI. One example is [24],

$$\cos\delta = \frac{\tan\theta_{23}}{\sin 2\theta_{12}\,\sin\theta_{13}}\left[\cos 2\theta^{\nu}_{12} + \left(\sin^2\theta_{12} - \cos^2\theta^{\nu}_{12}\right)\left(1 - \cot^2\theta_{23}\,\sin^2\theta_{13}\right)\right],$$

where θ_{12}, θ_{13}, θ_{23} are the neutrino mixing angles and δ is the Dirac CP-violating phase in the standard parametrisation of the neutrino mixing matrix [29], whereas θ^{ν}_{12} is a parameter fixed by the assumed underlying symmetry. In [24, 27] different mixing sum rules have been derived, and in [25–27] the phenomenological consequences of these sum rules have been studied. In [30] sum rules and predictions for $\cos\delta$ have been obtained from all possible types of residual symmetries in the charged lepton and neutrino sectors, and in [31] viability of these scenarios has been analysed in light of global neutrino oscillation data. In these studies it was assumed that the sum rule is exactly realised at low energy. However, as every quantity in quantum field theory, the mixing parameters get affected by renormalisation group running which has been recently updated and studied systematically in [32–34]. Those corrections can be particularly important in GUT scenarios where these sum rules can occur as well [35–37].

With the help of mixing sum rules one can examine currently viable flavour models based on different discrete symmetries and forecast the impact of measurements at future neutrino oscillation experiments [31, 38–44]. This strengthens further the physics programme of the long-baseline projects, such as DUNE, T2HK, ESSνSB, and the medium-baseline JUNO experiment, which are currently under construction or development. Mixing sum rules are particularly useful to distinguish between different flavour models which lead to distinct predictions for the Dirac CP-violating phase [24, 31] which is going to be measured at near-future long-baseline experiments. Moreover, since they also predict correlations among certain mixing angles, future precision measurements will help to further discriminate between different flavour models [43, 44]. Furthermore, leptonic mixing sum rules can be used to determine the experimental precision on the mixing parameters required to disentangle different models.

IV. SUMMARY

In conclusion, we have highlighted the importance of leptonic sum rules which lead to predictions testable at near-future neutrino experiments. Neutrino mass sum rules affect the effective Majorana mass in neutrinoless double beta decay as well as the other observables sensitive to the absolute neutrino mass scale. Leptonic mixing sum rules predict correlations between the three leptonic mixing angles and the CP-violating phases. As future long-baseline experiments (DUNE, T2HK, etc.) will determine the Dirac phase δ and together with current and future reactor experiments (Daya Bay, RENO, JUNO) provide precision measurements of the mixing angles, leptonic mixing sum rules will help to narrow down a broad class of flavour models based on discrete non-Abelian symmetries and scrutinise the models which survive the future constraints. The predictions of both classes of sum rules can be used to plan the stages of the respective experiments and provide guidance for the experimental programmes to identify the parameter regions of interest and the precision necessary to discriminate among different flavour models. Future detailed studies of sum rules are hence a great opportunity for model builders, phenomenologists and experimentalists to gain more insights into the mysteries hidden in the neutrino sector.

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