Snowmass LOI: Computing Neutrino Oscillations in Matter Efficiently

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(Dated: July 2, 2020)

Calculating neutrino oscillation probabilities in matter requires diagonalizing a matrix and is computationally expensive. Analytic approximations can significantly speed this process up and are now beginning to be implemented in experimental analysis chains for the standard three-flavor oscillation picture.

Frontiers: Primary: NF08 and CompF2. Secondary: NF01, NF02, NF03, TF

Overview A large fraction of the neutrino oscillation experiments performed today and in coming years involve the Wolfenstein matter effect [1] which alters the propagation of electron neutrinos depending on the density of the material and the energy of the neutrino. That is, while extracting the six parameters governing oscillations (θ_{12}, θ_{13}, θ_{23}, δ, Δm^2_{21}, and Δm^2_{31}) from a measurement of oscillations in vacuum is straightforward, in matter neutrinos propagate in a new basis that is related to the vacuum basis by the matter potential which depends on the neutrino energy. The matter effect plays a key role for long-baseline neutrino experiments, in particular NOvA [2] and DUNE [3] as well as T2K [4] and T2HK [5]. In addition, the matter effect dominates the physics of solar neutrino propagation in the sun via the MSW effect [1, 6] as well as the standard matter effect in the Earth when calculating the day-night asymmetry. In fact, there is a slight tension in Δm^2_{21} at the $\sim 2\,\sigma$ level between solar measurements which experience a significant matter effect and reactor measurements which have nearly no matter effect. Hyper-KamiokaNDE is expected to improve upon Super-KamiokaNDE's solar neutrino measurements [5, 7] and DUNE and JUNO should also have sensitivity to solar neutrinos [8, 9]. Finally, while the impact of the matter effect on ν_μ disappearance in atmospheric neutrinos is small, it does play an important role in ν_e appearance searches at Super-KamiokaNDE [10] as well as $\Delta m^2_{41} \sim 1$ eV^2 searches at IceCube [11].

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Computational Effort Calculating oscillation probabilities in matter requires finding the eigenvalues and diagonalizing 3×3 matrices (or larger in the case of sterile neutrinos); this process is computationally expensive. For example, the long-baseline accelerator experiment NOvA used 50,000,000 core hours for one of their long-baseline analyses [12]. A careful examination revealed that a substantial amount of that time was spent calculating oscillation probabilities. This can be understood in that there are a large number of systematic parameters that must be constrained, there is energy smearing between true neutrino energy and reconstructed energy, and because a Feldman-Cousins [13] procedure is employed to estimate confidence intervals. For each throw of the different parameters, the oscillation probability must be recalculated.

Exact Methods Numerous tools exist to investigate efficient means of performing such calculations in the context of neutrino oscillations. First, there are several fairly popular publicly available numerical codes that compute oscillation probabilities including matter effects up to machine precision in various different ways such as Prob3++ [14, 15], GLoBES [16–18], and nuSQuIDS [19]. The exact solution for neutrino oscillations in matter involves solving a cubic equation which contains a term $\cos(\frac{1}{3}\cos^{-1}(\cdots))$ [14, 20–24]. This term is unavoidable and does not trivially simplify. It is present in every aspect of the exact expressions which makes it extremely difficult to gain any useful physical understanding of the impact of the matter effect on neutrino oscillations.

Approximate Methods To address this term, many approximate expressions in the context of long-baseline oscillation probabilities have been developed over the years to address leveraging three main smallness parameters in different combinations and ways: 1) The ratio of Δm^2's, 2) θ_{13}, and 3) the matter potential $2EV_{\rm CC}/\Delta m^2_{31}$ [25–47]. A comparative review of many of these techniques was performed in ref. [48] which considered the precision of the various expressions in the context of DUNE and determined the computational time using public code [49]. This review was updated in ref. [42]. This comparison showed that approximate techniques are a factor of a few faster than the exact analytic expressions from ref. [20] (similar to the methods used in [14, 15] of Prob3++). Off-the-shelf linear algebra tools such as Eigen3 [50] for diagonalizing arbitrary 3×3 matrices are another order of magnitude slower than the exact analytic expression. Among those that have a satisfactory precision for long-baseline oscillations, three are fairly efficient computationally: refs. [34–36] with the most precise being ref. [36] (DMP hereafter). All three of these methods rely on a procedure of systematically changing the basis to optimally approximate the true basis and have several advantages over some other approximation schemes including the simultaneous calculation of all channels. DMP is not only precise enough for DUNE [51] but can also be extended to higher precision for only marginal additional computational cost. In addition, these techniques can be applied to new physics scenarios such as sterile neutrinos where a speed up over exact methods was also found [52].

Discussion Preliminary tests using the DMP expressions in NOvA's analysis chains show a factor of ~ 17 speed-up in the oscillation calculation itself consistent with ref. [48], which results in a factor of ~ 3 speed-up of statistics only fits for oscillation parameters. Deviations in the allowed regions of the oscillation parameters between using exact expressions and the approximate expressions were negligible [53]. Additionally, we note that the techniques used to develop these tools have also lead to other non-trivial results in mathematics [42, 54] which has consequences in any field that overlaps with linear algebra.

Conclusions In conclusion, we have highlighted the importance of carefully examining how neutrino oscillation probabilities in matter are calculated. Off-the-shelf linear algebra tools or the exact analytic expressions do work. However neither leads to particularly useful interpretation of the results. In addition, it has been shown that experimental analyses are computationally expensive with the oscillation probability as an important bottleneck. We now have approximate expressions that are both precise enough for current and upcoming experiments and an order of magnitude faster than previous methods leading to real computational gains in actual analyses. We hope these methods are further developed and further implemented in experimental analysis chains.

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