Snowmass2021 - Letter of Interest

Family Symmetry as a Possible Explanation for Lepton Masses and Mixing

NF Topical Groups:

■ (NF3) Beyond the Standard Model
■ (NF5) Neutrino properties
■ (TF11) Theory of neutrino physics

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At currently achievable energies, the Standard Model is a very good working theory. However, it is commonly believed that it is only an effective theory which at higher energies need to be modified. One of the signs of this state of things is a large number of free parameters that now need to be fitted from experiments. One of several proposals on how to restrict the number of free parameters in the SM is to introduce symmetry between Yukawa constants in Yukawa SM interaction in such a way that after spontaneous symmetry breaking get masses and mixing matrix parameters for quarks and leptons which are consistent with experience. Such symmetry is known in the literature as a flavor (family/horizontal) symmetry. Lepton sector and especially neutrino physics is an attractive area to search for such symmetry due to the so-called lepton mixing matrix. The imposition of a flavor symmetry on the leptonic part of the Yukawa Lagrangian has been discussed in [1, 2, 3]. In the SM, application of family symmetry is limited due to the Schur's lemma which implies that for three-dimensional mass matrices of charged leptons and neutrinos their diagonalization matrices are proportional to identity, thus the PMNS matrix becomes trivial. This drawback can be overcome in two ways. One approach is to break the family symmetry group by a scalar singlet, so-called flavon [4]. Despite many attempts, it has failed to reconstruct the PMNS matrix. In the second approach, a non-trivial mixing can be achieved by extending the Higgs sector by additional multiplets [5]. In this context, in the recent studies for the two Higgs doublet model were examined [6]. Finite, non-Abelian subgroups of the $U(3)$ group up to the order of 1025 were investigated. The results were utterly negative. Theoretical proposals assuming the existence of two Higgs fields are not the only possible and potentially experimentally verifiable space for applying the symmetry implementation, methodology proposed previously for 2HDM can be applied to any number of additional Higgs fields. On the other hand, such a theoretical framework can be also considered with the full 6×6 neutrino mixing matrix. In this way, a natural link with $3 + N$ Majorana neutrinos and CP-violating scenarios can be analyzed.

In the horizontal symmetry approach, a catalog of up to now considered groups is limited for two reasons. Firstly, the total number of non-Abelian groups of some given order rapidly increases for higher orders [7], factoring into strong growth of calculation complexity and resources for methods based on numerical analysis. Namely, the calculation of high dimensional non-Abelian groups is strongly time and memory consuming. On the other hand, more elegant analytical methods, based on group theory properties are complicated and time-consuming. Because of that quaternions can be considered as an alternative to up to now used numerical tools. Quaternions are substantially easier to work with as representations of individual rotations and their multiplications and a quaternion linear algebra offers a reduction in memory and arithmetic operations [8, 9]. Possibly a wider set of groups can be studied. Additionally, an approach with the use of quaternions can be also adopted for faster sifting of mixing matrices in dilation procedure, a crucial procedure to find new bounds on non-standard mixings in $3+N$ scenarios [10].

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