Snowmass 2021 Letter of Interest: Rare Kaon Decays $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0\nu\bar{\nu}$, $K_S \to \mu^+\mu^-$ and $K_L \to \pi^0\ell^+\ell^-$

Jason Aebischer[a], **Andrzej J. Buras**[b] **and Jacky Kumar**[c]

[a] Department of Physics, University of California at San Diego, La Jolla, CA 92093, USA
[b]TUM Institute for Advanced Study, Lichtenbergstr. 2a, D-85747 Garching, Germany
[c]Physique des Particules, Universite de Montreal, C.P. 6128, succ. centre-ville,
Montreal, QC, Canada H3C 3J7

We stress the importance of the precise measurements of the branching ratios for rare decays $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0\nu\bar{\nu}$, $K_S \to \mu^+\mu^-$ and $K_L \to \pi^0\ell^+\ell^-$ for the search of new physics (NP). In particular the correlations between these branching ratios on the one hand and their correlations with the ratio ε'/ε in $K_L \to \pi\pi$ decays, the CP-violating parameter ε_K and the $K^0 - \bar{K}^0$ mass difference ΔM_K, on the other hand, should help to disentangle the nature of possible NP.

Among the rare Kaon decays just listed $K^+ \to \pi^+\nu\bar{\nu}$ is CP conserving while $K_L \to \pi^0\nu\bar{\nu}$, $K_S \to \mu^+\mu^-$ and $K_L \to \pi^0\ell^+\ell^-$ proceed only in the presence of CP violation. The latter fact makes the search for these decays very important with the goal to find new sources of CP violation possibly responsible for the matter-antimatter asymmetry in the universe. A recent extensive review of these decays can be found in [1].

Within the SM these decays are loop-induced semileptonic FCNC processes, receiving only contributions from Z^0-penguin and box diagrams. A particular and very important virtue of $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ decays is their clean theoretical character. This is related to the fact that the low energy hadronic matrix elements required for the calculations of their branching ratios are just the matrix elements of quark currents between hadron states, which can be extracted assuming isospin symmetry from the leading (non-rare) semileptonic decay $K^+ \to \pi^0 e^+\nu$ that is very well measured. Isospin breaking and electroweak corrections are also known [2].

The case of $K_S \to \mu^+\mu^-$ and $K_L \to \pi^0\ell^+\ell^-$ is different as they are subject to long distance contributions. However, over the past years the understanding of the latter contributions has been improved by much [3–7]. This applies also to $K_L \to \mu^+\mu^-$ which already for many years bounded the estimates for the $K^+ \to \pi^+\nu\bar{\nu}$ rate in various NP scenarios, depending on whether NP contributions are dominated by left-handed or right-handed currents. On the other hand as stressed in [5] $K_L \to \pi^0\mu^+\mu^-$ and $K_L \to \pi^0 e^+e^-$ considered simultaneously offer a powerful test of scalar and pseudoscalar currents.

The investigation of these low energy rare decay processes in conjunction with their theoretical cleanliness allows to probe, albeit indirectly, high energy scales of the theory far beyond the reach of the LHC. They are also very sensitive to the values of the CKM parameters, in particular to V_{td} and $\mathrm{Im}\lambda_\mathrm{t} = \mathrm{Im}\mathrm{V}_\mathrm{ts}^*\mathrm{V}_\mathrm{td}$ so that the latter could in principle be extracted from precise measurements of the decay rates for $K^+ \to \pi^+\nu\bar{\nu}$ and $K_\mathrm{L} \to \pi^0\nu\bar{\nu}$, respectively. Moreover, the combination of these two decays offers one of the cleanest measurements of $\sin 2\beta$ [8]

with β being one of the angles of the Unitarity Triangle. However, the very fact that these processes are based on higher order electroweak effects implies that their branching ratios are expected to be very small and not easy to access experimentally.

As of 2020 one can look back at four decades of theoretical efforts to calculate the branching ratios for these decays within the SM. Among early calculations are [9, 10] in which QCD corrections were neglected. The first LO QCD corrections have been calculated in [11, 12] and the NLO ones in the 1990s [13–16]. Already the NLO calculations reduced significantly various renormalization scale uncertainties present at LO. Yet, in the last twenty years further progress has been made through the following calculations:

- NNLO QCD corrections to the charm contributions in $K^+ \to \pi^+\nu\bar{\nu}$: [17–19].
- Isospin breaking effects and non-perturbative effects: [2, 20].
- Complete NLO electroweak corrections to the charm quark contribution to $K^+ \to \pi^+\nu\bar{\nu}$: [21].
- Complete NLO electroweak corrections to the top quark contribution to $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$: [22].

As far as $K_{L,S} \to \mu^+\mu^-$ and $K_L \to \pi^0\ell^+\ell^-$ are concerned

- The NLO QCD calculations have been performed in [14, 16] and [23], respectively and the NNLO ones for $K_L \to \mu^+\mu^-$ in [24].
- The long-distance contributions have been investigated in [3–7].

Reviews of these decays can be found in [1, 25–29] and the power of them in testing energy scales as high as several hundreds of TeV has been demonstrated in [30]. This list of theoretical papers demonstrates very clearly the importance of these decays.

The SM predictions for $K \to \pi\nu\bar{\nu}$ decays are given as follows [31, 32]

$$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})_{\rm SM} = (8.5^{+1.0}_{-1.2}) \times 10^{-11}\,, \qquad \mathcal{B}(K_L \to \pi^0\nu\bar{\nu})_{\rm SM} = (3.2^{+1.1}_{-0.7}) \times 10^{-11}\,. \tag{1}$$

and for the remaining decays one finds [3–5, 32]:

$$\mathcal{B}(K_S \to \mu^+\mu^-)_{\rm SM} = (5.2 \pm 1.5) \times 10^{-12}\,, \quad \mathcal{B}(K_L \to \pi^0 e^+e^-)_{\rm SM} = 3.54^{+0.98}_{-0.85}(1.56^{+0.62}_{-0.49}) \times 10^{-11}\,,$$

$$\mathcal{B}(K_L \to \pi^0\mu^+\mu^-)_{\rm SM} = 1.41^{+0.28}_{-0.26}(0.95^{+0.22}_{-0.21}) \times 10^{-11}\,, \tag{2}$$

where for the $K_L \to \pi^0\ell^+\ell^-$ decays the numbers in parenthesis denote the destructive interference case.

On the experimental side the NA62 experiment at CERN is presently running and is expected to measure the $K^+ \to \pi^+\nu\bar{\nu}$ branching ratio with the precision of 10% by 2024, as described in [33], that would improve the accuracy of the previous measurement by a factor of five. The expected measurement of $K_L \to \pi^0\nu\bar{\nu}$ by KOTO at J-PARC [26, 34] should reach the SM level by 2024.

The most recent result for $K^+ \to \pi^+\nu\bar{\nu}$ from NA62 [35, 36] and the 90% confidence level (CL) upper bound on $K_L \to \pi^0\nu\bar{\nu}$ from KOTO [37] read respectively

$$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})_{\rm exp} = (11.0^{+4.0}_{-3.5} \pm 0.3) \times 10^{-11}\,, \qquad \mathcal{B}(K_L \to \pi^0\nu\bar{\nu})_{\rm exp} \le 3.0 \times 10^{-9}\,. \tag{3}$$

In their most recent status report [38] on $K_L \to \pi^0\nu\bar{\nu}$ the KOTO collaboration presented data on four candidate events in the signal region, finding

$$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})_{\text{KOTO}} = 2.1^{+2.0(+4.1)}_{-1.1(-1.7)} \times 10^{-9}\,, \tag{4}$$

at the 68 (95) % CL. The central value is by a factor of 65 above the central SM prediction and in fact violates the Grossman-Nir bound which at the 90% CL together with the present NA62 result for $K^+ \to \pi^+\nu\bar{\nu}$ amounts to 0.8×10^{-9}. Theoretical analyses of this interesting data can be found in [39–42].

As far as $K_S \to \mu^+\mu^-$ and $K_L \to \pi^0\ell^+\ell^-$ are concerned only upper bounds on them from LHCb on $K_S \to \mu^+\mu^-$ [43] and from KTeV on $K_L \to \pi^0\ell^+\ell^-$ [44, 45] are known

$$\begin{aligned} &\mathcal{B}(K_S \to \mu^+\mu^-)_{\text{LHCb}} < 0.8(1.0) \times 10^{-9}\,, \quad \mathcal{B}(K_L \to \pi^0 e^+e^-)_{\text{KTeV}} < 28 \times 10^{-11}\,,\\ &\mathcal{B}(K_L \to \pi^0\mu^+\mu^-)_{\text{KTeV}} < 38 \times 10^{-11}\,, \end{aligned} \tag{5}$$

The main theoretical uncertainties in the SM predictions for $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ stem from the CKM parameters, in particular $|V_{cb}|$ in the case of $K^+ \to \pi^+\nu\bar{\nu}$ and both $|V_{cb}|$ and $|V_{ub}|$ in the case of $K_L \to \pi^0\nu\bar{\nu}$. In the case of $K^+ \to \pi^+\nu\bar{\nu}$ an improved estimate of long distance effects in charm contributions is desirable. Lattice QCD should be helpful in this respect and in fact first steps in this direction have been made by the RBC-UKQCD collaboration [46, 47]. It is expected that in the second half of 2020 the errors in (1) will be decreased below 0.5 and 0.3 for $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$, respectively. This precision is also expected on the experimental side, in particular for $K^+ \to \pi^+\nu\bar{\nu}$.

While precise measurements of all these channels constitute important tests of the SM, their particular role in the coming years will be in the context of the search for NP through deviations from SM predictions. An extendable list of possible avenues for the exploration of these decays is as follows:

- Correlations between these decays among themselves with constraints from ε_K as already investigated in the past in particular in [8,48,49] but also with ε'/ε and ΔM_K investigated recently [50, 51]. Not only an efficient search for new CP-violating phases can be made in this manner but also getting a handle on right-handed currents is possible.

- Of importance are also correlations between these decays and rare B-meson decays, in particular with $B \to K(K^*)\nu\bar{\nu}$ [52–54] but also with $B_{s,d} \to \mu^+\mu^-$.

- These decays have been analyzed in numerous SM extensions with a detailed review in the recent book [1].

  Here we mention supersymmetric models [55–57], selected models with Z' exchanges [58–60], induced FCNCs mediated by the Z boson [61, 62] as well as the analyses of these decays in the context of the violation of lepton flavour universality (LFUV) should be mentioned. These are within models with vector-like quarks [32], leptoquark models [63, 64] and also in $K \to \pi\nu\bar{\nu}$ decays [65] through the presence of the 3rd generation neutrinos. The tests of LFUV can also be made through $K \to \pi\ell\ell'$ and $K \to \ell\ell'$ [66].

  Finally the presence of lepton number violating decays $K \to \pi\nu\nu$ would signal the Majorana character of the neutrinos. As analysed recently in [67] these decays could be distinguished from $K \to \pi\nu\bar{\nu}$ through kinematical distributions. They are sensitive to scalar currents and to NP scales as high as 20 TeV. Although neutrino-less β decay probes higher scales, it is limited to first generations of leptons and quarks only, while the rare Kaon decays in question open up a window to different quark and neutrino flavours.

References

[1] A. J. Buras, *Gauge Theory of Weak Decays.* Cambridge University Press, 6, 2020.

[2] F. Mescia and C. Smith, *Improved estimates of rare K decay matrix-elements from $K_{\ell 3}$ decays*, *Phys. Rev.* **D76** (2007) 034017, [`arXiv:0705.2025`].

[3] G. Isidori and R. Unterdorfer, *On the short-distance constraints from $K_{L,S} \to \mu^+\mu^-$*, *JHEP* **01** (2004) 009, [`hep-ph/0311084`].

[4] G. D'Ambrosio and T. Kitahara, *Direct CP Violation in $K \to \mu^+\mu^-$*, *Phys. Rev. Lett.* **119** (2017), no. 20 201802, [`arXiv:1707.06999`].

[5] F. Mescia, C. Smith, and S. Trine, *$K_L \to \pi^0 e^+e^-$ and $K_L \to \pi^0\mu^+\mu^-$: A binary star on the stage of flavor physics*, *JHEP* **08** (2006) 088, [`hep-ph/0606081`].

[6] G. D'Ambrosio, D. Greynat, and M. Knecht, *On the amplitudes for the CP-conserving $K^\pm(K_S) \to \pi^\pm(\pi^0)\ell^+\ell^-$ rare decay modes*, *JHEP* **02** (2019) 049, [`arXiv:1812.00735`].

[7] G. D'Ambrosio, D. Greynat, and M. Knecht, *Matching long and short distances at order $\mathcal{O}(\alpha_s)$ in the form factors for $K \to \pi\ell^+\ell^-$*, *Phys. Lett. B* **797** (2019) 134891, [`arXiv:1906.03046`].

[8] G. Buchalla and A. J. Buras, *$\sin 2\beta$ from $K \to \pi\nu\bar{\nu}$*, *Phys. Lett.* **B333** (1994) 221–227, [`hep-ph/9405259`].

[9] M. Gaillard and B. W. Lee, *Rare Decay Modes of the K-Mesons in Gauge Theories*, *Phys. Rev.* **D10** (1974) 897.

[10] T. Inami and C. Lim, *Effects of Superheavy Quarks and Leptons in Low-Energy Weak Processes $K_L \to \mu^+\mu^-$, $K^+ \to \pi^+\nu\bar{\nu}$ and $K^0 - \bar{K}^0$*, *Prog. Theor. Phys.* **65** (1981) 297.

[11] J. R. Ellis and J. S. Hagelin, *Constraints on Light Particles from Kaon Decays*, *Nucl. Phys.* **B217** (1983) 189–214.

[12] C. Dib, I. Dunietz, and F. J. Gilman, *Strong Interaction Corrections to the Decay $K \to \pi$ Neutrino Anti-neutrino for Large M(t)*, *Mod. Phys. Lett.* **A6** (1991) 3573–3582.

[13] G. Buchalla and A. J. Buras, *QCD corrections to rare K and B decays for arbitrary top quark mass*, *Nucl. Phys.* **B400** (1993) 225–239.

[14] G. Buchalla and A. J. Buras, *The rare decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \mu^+\mu^-$ beyond leading logarithms*, *Nucl. Phys.* **B412** (1994) 106–142, [`hep-ph/9308272`].

[15] M. Misiak and J. Urban, *QCD corrections to FCNC decays mediated by Z penguins and W boxes*, *Phys. Lett.* **B451** (1999) 161–169, [`hep-ph/9901278`].

[16] G. Buchalla and A. J. Buras, *The rare decays* $K \to \pi\nu\bar{\nu}$*,* $B \to X\nu\bar{\nu}$ *and* $B \to \ell^+\ell^-$*: An Update*, *Nucl. Phys.* **B548** (1999) 309–327, [`hep-ph/9901288`].

[17] A. J. Buras, M. Gorbahn, U. Haisch, and U. Nierste, *The rare decay* $K^+ \to \pi^+\nu\bar{\nu}$ *at the next-to-next-to-leading order in QCD*, *Phys. Rev. Lett.* **95** (2005) 261805, [`hep-ph/0508165`].

[18] A. J. Buras, M. Gorbahn, U. Haisch, and U. Nierste, *Charm quark contribution to* $K^+ \to \pi^+\nu\bar{\nu}$ *at next-to-next-to-leading order*, *JHEP* **11** (2006) 002, [`hep-ph/0603079`].

[19] M. Gorbahn and U. Haisch, *Effective Hamiltonian for non-leptonic* $|\Delta F| = 1$ *decays at NNLO in QCD*, *Nucl. Phys.* **B713** (2005) 291–332, [`hep-ph/0411071`].

[20] G. Isidori, F. Mescia, and C. Smith, *Light-quark loops in* $K \to \pi\nu\bar{\nu}$, *Nucl. Phys.* **B718** (2005) 319–338, [`hep-ph/0503107`].

[21] J. Brod and M. Gorbahn, *Electroweak Corrections to the Charm Quark Contribution to* $K^+ \to \pi^+\nu\bar{\nu}$, *Phys. Rev.* **D78** (2008) 034006, [`arXiv:0805.4119`].

[22] J. Brod, M. Gorbahn, and E. Stamou, *Two-Loop Electroweak Corrections for the* $K \to \pi\nu\bar{\nu}$ *Decays*, *Phys. Rev.* **D83** (2011) 034030, [`arXiv:1009.0947`].

[23] A. J. Buras, M. E. Lautenbacher, M. Misiak, and M. Munz, *Direct CP violation in* $K_L \to \pi^0 e^+ e^-$ *beyond leading logarithms*, *Nucl. Phys.* **B423** (1994) 349–383, [`hep-ph/9402347`].

[24] M. Gorbahn and U. Haisch, *Charm quark contribution to* $K_L \to \mu^+\mu^-$ *at next-to-next-to-leading order*, *Phys. Rev. Lett.* **97** (2006) 122002, [`hep-ph/0605203`].

[25] A. J. Buras, F. Schwab, and S. Uhlig, *Waiting for precise measurements of* $K^+ \to \pi^+\nu\bar{\nu}$ *and* $K_L \to \pi^0\nu\bar{\nu}$, *Rev. Mod. Phys.* **80** (2008) 965–1007, [`hep-ph/0405132`].

[26] T. Komatsubara, *Experiments with K-Meson Decays*, *Prog. Part. Nucl. Phys.* **67** (2012) 995–1018, [`arXiv:1203.6437`].

[27] A. J. Buras and J. Girrbach, *Towards the Identification of New Physics through Quark Flavour Violating Processes*, *Rept. Prog. Phys.* **77** (2014) 086201, [`arXiv:1306.3775`].

[28] M. Blanke, *New Physics Signatures in Kaon Decays*, *PoS* **KAON13** (2013) 010, [`arXiv:1305.5671`].

[29] C. Smith, *Rare K decays: Challenges and Perspectives*, `arXiv:1409.6162`.

[30] A. J. Buras, D. Buttazzo, J. Girrbach-Noe, and R. Knegjens, *Can we reach the Zeptouniverse with rare* K *and* $B_{s,d}$ *decays?*, *JHEP* **1411** (2014) 121, [`arXiv:1408.0728`].

[31] A. J. Buras, D. Buttazzo, J. Girrbach-Noe, and R. Knegjens, $K^+ \to \pi^+\nu\bar{\nu}$ *and* $K_L \to \pi^0\nu\bar{\nu}$ *in the Standard Model: status and perspectives*, *JHEP* **11** (2015) 033, [`arXiv:1503.02693`].

[32] C. Bobeth, A. J. Buras, A. Celis, and M. Jung, *Patterns of Flavour Violation in Models with Vector-Like Quarks*, *JHEP* **04** (2017) 079, [`arXiv:1609.04783`].

[33] A. Ceccucci, *Review and Outlook on Kaon Physics*, *Acta Phys. Polon.* **B49** (2018) 1079–1086.

[34] **KOTO** Collaboration, K. Shiomi, $K_L^0 \to \pi^0 \nu\bar{\nu}$ *at KOTO*, in *8th International Workshop on the CKM Unitarity Triangle (CKM 2014) Vienna, Austria, September 8-12, 2014*, 2014. `arXiv:1411.4250`.

[35] **NA62** Collaboration, E. Cortina Gil et al., *An investigation of the very rare* $K^+ \to \pi^+ \nu\bar{\nu}$ *decay*, `arXiv:2007.08218`.

[36] R. Marchevski, " New result on the search for the $K^+ \to \pi^+ \nu\bar{\nu}$ decay at the NA62 experiment at CERN." ICHEP, Prague, 28 July-6 August, 2020.

[37] **KOTO** Collaboration, J. Ahn et al., *Search for the* $K_L \to \pi^0 \nu\bar{\nu}$ *and* $K_L \to \pi^0 X^0$ *decays at the J-PARC KOTO experiment*, *Phys. Rev. Lett.* **122** (2019), no. 2 021802, [`arXiv:1810.09655`].

[38] S. Shinohara, "Search for the rare decay $K_L \to \pi^0 \nu\bar{\nu}$ at J-PARC KOTO experiment." KAON2019, Perugia, Italy, 10-13 September, 2019.

[39] T. Kitahara, T. Okui, G. Perez, Y. Soreq, and K. Tobioka, *New physics implications of recent search for* $K_L \to \pi^0 \nu\bar{\nu}$ *at KOTO*, `arXiv:1909.11111`.

[40] X.-G. He, X.-D. Ma, J. Tandean, and G. Valencia, *Evading the Grossman-Nir bound with* $\Delta I = 3/2$ *new physics*, `arXiv:2005.02942`.

[41] X.-G. He, X.-D. Ma, J. Tandean, and G. Valencia, *Breaking the Grossman-Nir Bound in Kaon Decays*, *JHEP* **04** (2020) 057, [`arXiv:2002.05467`].

[42] K. Fuyuto, W.-S. Hou, and M. Kohda, *Loophole in* $K \to \pi \nu\bar{\nu}$ *Search and New Weak Leptonic Forces*, *Phys. Rev. Lett.* **114** (2015) 171802, [`arXiv:1412.4397`].

[43] **LHCb** Collaboration, R. Aaij et al., *Improved limit on the branching fraction of the rare decay* $K_S^0 \to \mu^+\mu^-$, *Eur. Phys. J.* **C77** (2017), no. 10 678, [`arXiv:1706.00758`].

[44] **KTEV** Collaboration, A. Alavi-Harati et al., *Search for the Decay* $K_L \to \pi^0 \mu^+\mu^-$, *Phys. Rev. Lett.* **84** (2000) 5279–5282, [`hep-ex/0001006`].

[45] **KTeV** Collaboration, A. Alavi-Harati et al., *Search for the rare decay* $K_L \to \pi^0 e^+ e^-$, *Phys. Rev. Lett.* **93** (2004) 021805, [`hep-ex/0309072`].

[46] **RBC, UKQCD** Collaboration, N. H. Christ, X. Feng, A. Portelli, and C. T. Sachrajda, *Prospects for a lattice computation of rare kaon decay amplitudes II* $K \to \pi \nu\bar{\nu}$ *decays*, *Phys. Rev.* **D93** (2016), no. 11 114517, [`arXiv:1605.04442`].

[47] **RBC, UKQCD** Collaboration, N. H. Christ, X. Feng, A. Portelli, and C. T. Sachrajda, *Lattice QCD study of the rare kaon decay* $K^+ \to \pi^+ \nu\bar{\nu}$ *at a near-physical pion mass*, *Phys. Rev. D* **100** (2019), no. 11 114506, [`arXiv:1910.10644`].

[48] M. Blanke, *Insights from the Interplay of* $K \to \pi \nu\bar{\nu}$ *and* ϵ_K *on the New Physics Flavour Structure*, *Acta Phys.Polon.* **B41** (2010) 127, [`arXiv:0904.2528`].

[49] A. J. Buras, D. Buttazzo, and R. Knegjens, $K \to \pi \nu\bar{\nu}$ *and* ϵ'/ϵ *in Simplified New Physics Models*, *JHEP* **11** (2015) 166, [`arXiv:1507.08672`].

[50] A. J. Buras, *New physics patterns in* ε'/ε *and* ε_K *with implications for rare kaon decays and* ΔM_K, *JHEP* **04** (2016) 071, [`arXiv:1601.00005`].

[51] J. Aebischer, A. J. Buras, and J. Kumar, *Another SMEFT Story: Z' Facing New Results on ε'/ε, ΔM_K and $K \to \pi\nu\bar{\nu}$*, `arXiv:2006.01138`.

[52] A. J. Buras and R. Fleischer, *Bounds on the unitarity triangle,* $\sin 2\beta$ *and $K \to \pi\nu\bar{\nu}$ decays in models with minimal flavor violation*, *Phys. Rev.* **D64** (2001) 115010, [`hep-ph/0104238`].

[53] W. Altmannshofer, A. J. Buras, D. M. Straub, and M. Wick, *New strategies for New Physics search in $B \to K^*\nu\bar{\nu}$, $B \to K\nu\bar{\nu}$ and $B \to X_s\nu\bar{\nu}$ decays*, *JHEP* **04** (2009) 022, [`arXiv:0902.0160`].

[54] A. J. Buras, J. Girrbach-Noe, C. Niehoff, and D. M. Straub, *$B \to K^{(*)}\nu\bar{\nu}$ decays in the Standard Model and beyond*, *JHEP* **1502** (2015) 184, [`arXiv:1409.4557`].

[55] A. J. Buras, T. Ewerth, S. Jager, and J. Rosiek, *$K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ decays in the general MSSM*, *Nucl. Phys.* **B714** (2005) 103–136, [`hep-ph/0408142`].

[56] G. Isidori, F. Mescia, P. Paradisi, C. Smith, and S. Trine, *Exploring the flavour structure of the MSSM with rare K decays*, *JHEP* **08** (2006) 064, [`hep-ph/0604074`].

[57] T. Blazek and P. Matak, *Left–left squark mixing, $K^+ \to \pi^+\nu\bar{\nu}$ and minimal supersymmetry with large tanβ*, *Int. J. Mod. Phys. A* **29** (2014), no. 27 1450162, [`arXiv:1410.0055`].

[58] A. J. Buras, F. De Fazio, J. Girrbach, and M. V. Carlucci, *The Anatomy of Quark Flavour Observables in 331 Models in the Flavour Precision Era*, *JHEP* **1302** (2013) 023, [`arXiv:1211.1237`].

[59] A. J. Buras, F. De Fazio, and J. Girrbach, *The Anatomy of Z' and Z with Flavour Changing Neutral Currents in the Flavour Precision Era*, *JHEP* **1302** (2013) 116, [`arXiv:1211.1896`].

[60] J. Aebischer, A. J. Buras, M. Cerdá-Sevilla, and F. De Fazio, *Quark-lepton connections in Z mediated FCNC processes: gauge anomaly cancellations at work*, *JHEP* **02** (2020) 183, [`arXiv:1912.09308`].

[61] C. Bobeth, A. J. Buras, A. Celis, and M. Jung, *Yukawa enhancement of Z-mediated new physics in $\Delta S = 2$ and $\Delta B = 2$ processes*, *JHEP* **07** (2017) 124, [`arXiv:1703.04753`].

[62] M. Endo, T. Kitahara, and D. Ueda, *SMEFT top-quark effects on $\Delta F = 2$ observables*, *JHEP* **07** (2019) 182, [`arXiv:1811.04961`].

[63] C. Bobeth and A. J. Buras, *Leptoquarks meet ε'/ε and rare Kaon processes*, *JHEP* **02** (2018) 101, [`arXiv:1712.01295`].

[64] S. Fajfer, N. Konik, and L. Vale Silva, *Footprints of leptoquarks: from $R_{K^{(*)}}$ to $K \to \pi\nu\bar{\nu}$*, *Eur. Phys. J.* **C78** (2018), no. 4 275, [`arXiv:1802.00786`].

[65] M. Bordone, D. Buttazzo, G. Isidori, and J. Monnard, *Probing Lepton Flavour Universality with $K \to \pi\nu\bar{\nu}$ decays*, *Eur. Phys. J.* **C77** (2017), no. 9 618, [`arXiv:1705.10729`].

[66] A. Crivellin, G. D'Ambrosio, M. Hoferichter, and L. C. Tunstall, *Violation of lepton flavor and lepton flavor universality in rare kaon decays*, *Phys. Rev.* **D93** (2016), no. 7 074038, [`arXiv:1601.00970`].

[67] F. F. Deppisch, K. Fridell, and J. Harz, *Probing lepton number violating interactions in rare kaon decays*, `arXiv:2009.04494`.