Study of pion and eta decays

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September 2020

The neutral pion is the lightest meson and, as such, it plays a significant role in the study of low-energy properties of the strong nuclear force. The π^0 decays almost instantaneously to two photons due to the electromagnetic interaction. The description of this and other related π^0 decays involves a π^0–γ^*–γ^* vertex, which is described by the so-called π^0 (electromagnetic) transition form factor (TFF). In general, a particle form factor encodes the information about particle structural properties and interactions with other particles, while leaving out the details about the underlying dynamics. In the case of π^0, the TFF describes the electromagnetic structure of the neutral pion and, within the Standard Model (SM), it stems from quantum chromodynamics.

By measuring properties of various π^0 decay channels such as the rare decay $\pi^0 \to e^+e^-$ and the Dalitz decay $\pi^0 \to e^+e^-\gamma$, one can probe the neutral-pion structure. This is very important in the context of another physical quantity involving the π^0 TFF — the anomalous magnetic moment of the muon $a_\mu = (g-2)_\mu$. It has served for many years as an important test of the SM: Currently, there is a discrepancy of 3.6 standard deviations between the SM theory expectation and the experimental result, which is generally considered as one of the most promising signs of new physics beyond the SM [1, 2]. The π^0 TFF in the space-like momentum region enters the so-called hadronic light-by-light (HLbL) scattering contribution to a_μ. The overall size of the HLbL contribution shows that a large part of the total prediction uncertainty comes from this contribution, the dominant part of which comes from the π^0 exchange. And here, different theoretical models agree only within 25 %.

An important experimental information used as an input to the π^0-TFF modeling is the slope of the form factor — the linear coefficient in the normalized transferred-momentum-squared expansion of its singly virtual version. The most precise model-independent measurement of the slope parameter to date was carried out by the NA62 experiment, analysing $\pi^0 \to e^+e^-\gamma$ decays from the 2007 data set [3]. The precision of the measurement was limited by its statistical uncertainty, which motivates a new model-independent measurement of the TFF slope exploiting the large data set of K^+ decays collected by the NA62 experiment during 2016–2018.

The π^0 TFF also enters the calculation of the $\pi^0 \to e^+e^-$ decay rate, with a persisting tension between theoretical predictions based on the SM and the KTeV E799-II experimental result [4]. When calculating this process theoretically, it is essential to include both real and virtual radiative corrections. The former play a crucial role in the analysis since $\pi^0 \to e^+e^-$ decay with an extra radiative photon is experimentally indistinguishable from a $\pi^0 \to e^+e^-\gamma$ decay. Recent calculations are roughly 2 stadard deviations away from experiment [5, 6]. In this regard, the study of the Dalitz decay was done in Refs. [7, 8, 9]. Moreover, the π^0 Dalitz decay is commonly used for normalization in the π^0-rare-decay and other measurements in the kaon sector, so its precise description is fundamental. The knowledge of the slope (representing the only relevant hadronic parameter) together with the radiative corrections can be translated into very precise predictions for the Dalitz-decay and two-photon-decay branching ratios [10].

Note that one can in principle easily extend the above decays into corresponding η decays (see e.g. Refs. [11, 12]). As explained in the following paragraph, their knowledge together with the weak charged decays is also essential for the precise calculations of the π^0 processes.

To demonstrate it, let us discuss $\pi^0 \to \gamma\gamma$ in more detail. This decay was crucial in establishing the role of the anomaly for the gauge theory. Without the anomaly, there was a long-standing puzzle how to overcome the implication of the Sutherland theorem, which stated that this process should be suppressed with respect to its actual measured value by a power of $m_\pi^2/(1\,\mathrm{GeV}^2)$. New experiments PrimEx and PrimeEx-II [13, 14] with a total uncertainty of 1.5 % motivate to understand these Sutherland contributions even better. Presumably, accidental non-existence of the leading logarithm (i.e. a term $\propto m_\pi^2 \log m_\pi^2$) [15] leads to a necessity to calculate even higher orders in m_π, i.e., in the language of the chiral perturbation theory (ChPT), one needs to calculate this process up to NNLO. This was performed for the two-flavour ChPT in Ref. [16] with a remarkably simple analytic result when rewritten using the so-called modified counting:

$$\begin{aligned} A_{\mathrm{NNLO}}^{\mathrm{mod}} = \frac{e^2}{F_\pi}\Bigg\{ & \frac{1}{4\pi^2} - \frac{64}{3} m_\pi^2 C_7^{Wr} + \frac{1}{16\pi^2}\frac{m_d - m_u}{m_s}\Big[1 - \frac{3}{2}\frac{m_\pi^2}{16\pi^2 F_\pi^2} L_\pi\Big] \\ & + 32B(m_d - m_u)\Bigg[\frac{4}{3} C_7^{Wr} + 4C_8^{Wr}\Big(1 - 3\frac{m_\pi^2}{16\pi^2 F_\pi^2} L_\pi\Big) \\ & - \frac{1}{16\pi^2 F_\pi^2}\Big(3L_7^r + L_8^r - \frac{1}{512\pi^2}(L_K + \frac{2}{3}L_\eta)\Big)\Bigg] - \frac{1}{24\pi^2}\left(\frac{m_\pi^2}{16\pi^2 F_\pi^2} L_\pi\right)^2 \Bigg\}. \end{aligned}$$

Apart from the leading order and logarithms, one ends up with two low-energy constants (LECs) C_7^W and C_8^W, on which the calculation of the $\pi^0 \to \gamma\gamma$ process depends. The former can be set using the resonance phenomenology and the latter can be obtained from the decay $\eta \to \gamma\gamma$: Here, we have similar dependence on C_7^W and C_8^W as for the π^0 decay. However, this process is known only at the NLO and, again, it contains no leading logarithms. The first attempt to calculate higher-order logarithms can be found in Ref. [17]. Note also an essential dependence on F_π — the pion decay constant. The value of this fundamental order parameter of the spontaneous chiral symmetry breaking can be set from the process $\pi^\pm \to \mu^\pm \nu_\mu(\gamma)$. The width of this weak charged-pion decay is $\Gamma \propto G_F^2 V_{ud}^2 F_\pi^2 m_\mu^2 m_\pi$ and leads to $F_\pi = 92.2(1)$. This determination is based on the SM, which includes the standard $V - A$ interaction. However, one can assume models beyond the SM, as e.g. in Ref. [18] where the existence of right-handed current was proposed. Thus, in principle, we have to distinguish $\hat{F}_\pi$ obtained from the weak decay and F_π, the parameter in the ChPT. Schematically,

$$F_\pi^2 = \hat{F}_\pi^2(1+\epsilon)\,, \quad \epsilon \propto V_{\mathrm{R}}^{ud}/V_{\mathrm{L}}^{ud}\,.$$

If turned around and the theoretical prediction for $\pi^0 \to \gamma\gamma$ is used for the extraction of F_π, we get a rough estimate $\epsilon \approx (3-4)\,\%$.

In summary, it is vital to deepen and extend our knowledge regarding the π^0/η–γ^*–γ^* vertex. This includes theoretical studies of pion decays (mainly $\pi^0 \to \gamma\gamma$, $\pi^0 \to e^+e^-\gamma$, $\pi^0 \to e^+e^-$, π_{l2}) and related eta decays. The interplay of theoretical studies (including the model building beyond the SM) and rich experimental program in this area (especially $g-2$ at FermiLab, NA62 at CERN and π^0 and η program at JLab) remains crucial for achieving our objectives.

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