Snowmass2021 - Letter of Interest

Precise Lattice QCD calculations of kaon and pion decay parameters and first-row CKM unitarity tests

Snowmass Topical Groups:

■ (RF2) Weak Decays of Strange and Light Quarks
■ (TF05) Lattice gauge theory
■ (EF05) QCD and Strong Interactions: Precision QCD
■ (CompF2) Theoretical calculations and simulation

Contact Information:
Aida X. El-Khadra (Department of Physics and ICASU, University of Illinois at Urbana-Champaign) [email]: axk@illinois.edu
Elvira Gámiz (CAFPE & Departamento de Física Teórica y del Cosmos, Universidad de Granada, Spain) [email]: megamiz@ugr.es

Authors List (alphabetical):
A. Bazavov (Department of Phyics, Michigan State University)
C. Bernard (Department of Physics, Washington University)
C. DeTar (Department of Physics and Astronomy, University of Utah)
A.X. El-Khadra (Department of Physics and ICASU, University of Illinois at Urbana-Champaign)
E. Gámiz (CAFPE & Departamento de Física Teórica y del Cosmos, Universidad de Granada, Spain)
Z. Gelzer (Department of Physics, University of Illinois)
S. Gottlieb (Department of Physics, Indiana University)
U.M. Heller (American Physical Society, New York)
W.I. Jay (Theoretical Physics Department, Fermilab)
A.S. Kronfeld (Theoretical Physics Department, Fermilab)
J. Laiho (Department of Physics, Syracuse University)
P.B. Mackenzie (Theoretical Physics Department, Fermilab)
E.T. Neil (Department of Physics, University of Colorado)
R. Sugar (Department of Physics, University of California, Santa Barbara)
J.N. Simone (Theoretical Physics Department, Fermilab)
D. Toussaint (Physics Department, University of Arizona)
R.S. Van de Water (Theoretical Physics Department, Fermilab)
A. Vaquero (Department of Physics and Astronomy, University of Utah)

High-precision tests of the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, as predicted by the Standard Model (SM), are an important component of the current flavor physics program. They allow both to search for evidence of physics beyond the Standard Model (BSM) and to put important constraints on the scale of the allowed new physics (NP) — see Ref. [1] and references therein. In particular, first-row unitarity, which requires that $\Delta_u \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1$ vanishes, is currently the most precisely tested condition. It allows us to test vector and axial-vector couplings to be SM or BSM.

At the current level of precision, the contribution from $|V_{ub}|$ is negligible, and SM tests through the first-row unitarity relation or through the universality of the Cabibbo angle [2] involve the high precision extraction of $|V_{ud}|$ and $|V_{us}|$, which contribute at comparable levels to the error budget. The most precise determination of $|V_{ud}|$ is from superallowed beta decays, while $|V_{us}|$ is determined from experimental measurements of leptonic or semileptonic kaon decay rates together with nonperturbative inputs obtained from lattice QCD for the semileptonic form factor $f_+^{K\pi}(0)$ or leptonic decay constant f_{K^+}. In the case of $|V_{us}|$ determinations from leptonic decay data, using the ratio of kaon and pion leptonic decay rates together with the ratio of decay constants f_{K^+}/f_{π^+} to obtain $|V_{us}|/|V_{ud}|$ yields slightly more precise results. A combination of semileptonic and leptonic decay rates of kaons and pions, together with the corresponding lattice inputs, can then also yield an alternative first-row CKM test, which avoids the use of nuclear decays.

In the past decade, through dedicated, world-wide effort, lattice QCD (LQCD) has been established as a reliable tool for providing such non-perturbative inputs at high precision. Indeed, LQCD calculations of the form factors and decay constants have reached 0.2% and 0.16% level uncertainties, respectively [3–5].

In addition to these high precision LQCD calculations of the non-perturbative inputs, there are recent determinations of radiative corrections, both universal and structure dependent, entering the extraction of $|V_{ud}|$ from superallowed beta decays. Together, these developments are responsible for current tensions in the SM unitarity relation at a 2–5σ level — see, for example, Ref. [2] and references therein. There is also an internal tension between leptonic and semileptonic determinations of $|V_{us}|$ within the SM using $|V_{ud}|$ from superallowed beta decay as an input [5]. Further, the determination of $|V_{ud}|$ from superallowed beta decay is controversial, due to internal tensions between different determinations of the radiative corrections mentioned above.

Improvements in the LQCD inputs will be crucial in elucidating the origins of current tensions and providing as many independent and high precision determinations of $|V_{ud}|$ and $|V_{us}|$ as possible. The level of precision reached by LQCD calculations of light-meson form factors and decay constants makes it necessary to include higher-order effects such as QED and strong isospin-breaking corrections directly in the simulations, instead of estimating them using phenomenological considerations. We are also able to attack more challenging calculations of weak processes mediated beyond tree-level, such as rare decays or some of the structure-dependent contributions to the radiative corrections entering in the extraction of $|V_{ud}|$.

Our ongoing kaon and pion projects will contribute to this program in the near future. We are currently working on refining our analysis of the pion and kaon decay constants [3], which will allow us to include the statistical and systematic correlations between them and the kaon form factor, enabling a more precise test of first-row CKM unitarity without nuclear physics inputs. In view of the current controversy in the $|V_{ud}|$ determination from superallowed beta decays, that has been traditionally used to fix the value of the pion decay constant f_π, it is important to move to a lattice scale setting that does not involve f_π as the external input. This would also allow us to obtain f_π and f_K separately (not just the ratio f_K/f_π), which could be used for BSM analyses, as well as providing a complete description of pion leptonic decays.

In addition, over the next decade, we expect the following important developments to further reduce the theoretical uncertainties on semileptonic and leptonic K and π processes, as well as to extend the landscape of high precision LQCD calculations in this field:

- **QED and strong isospin breaking effects**: At present, the isospin breaking corrections needed in the determinations of $|V_{us}|$ and $|V_{ud}|$ from kaon and pion decays are estimated phenomenologically [6], and the resulting uncertainties are not completely quantified. This includes strong isospin breaking, virtual, and radiative QED effects. Hence, in order to further improve the CKM determinations, these effects must be calculated with lattice methods. This is particularly important for the determination of $|V_{us}|$ from semileptonic kaon decay, as the phenomenologically evaluated radiative corrections carry model dependence, and hence may not be fully quantified. Fortunately, a variety of methods already exist to compute isospin breaking corrections to hadronic quantities using lattice techniques [5], and methods developed in recent years also enable lattice calculations of radiative corrections. For example, first results for isospin corrections of leptonic decay amplitudes are presented in Ref. [7], while first calculations of radiative leptonic decay rates are shown in Refs. [8, 9]. More work is certainly needed and we plan to build on our ongoing projects on isospin corrections to the muon $g-2$ to develop calculations of these corrections and of the related radiative rates. This effort will also be beneficial to our B- and D-meson decay projects described in our companion heavy-flavor letter of intent.

- **Rare kaon decay processes:** Rare kaon decays are very promising channels in the search for New Physics, especially considering the active, ongoing and future experimental programs at CERN (NA62, LHCb, KLEVER) and J-PARC (KOTO, KOTO-II), see, for example, Refs. [10, 11]. However, in rare kaon decays, long-distance effects are either a dominant contribution (as in $K \to \pi\ell\ell$) or a small but important correction that should be better quantified (as in $K \to \pi\nu\bar{\nu}$). Although challenging for LQCD, recent theoretical and technological developments have made possible the first exploratory studies of some of these contributions [12, 13]. In the future, LQCD can become a unique tool for providing the needed inputs, and we plan to use the infrastructure we are developing for QED corrections and for inclusive decay rates to extend our ongoing kaon program to include them. New computations can also provide refinements to our current semileptonic kaon form factor project by including the full q^2 dependence as well as the tensor form factor relevant for the short-distance contributions to rare kaon decay amplitudes.

- **Radiative corrections relevant to nuclear extractions of** $|V_{ud}|$: LQCD calculations can also help to disentangle the internal tensions in the determination of radiative corrections to the extraction of $|V_{ud}|$. The first realistic calculation of the γW box correction to pion semileptonic decay, a dominant source of uncertainty, has been already presented in Ref. [14]. The techniques developed can also be generalized to cover other radiative corrections relevant for $|V_{ud}|$ extractions.

- $|V_{us}|$ **from inclusive strange hadronic** τ **decays**: An alternative extraction of $|V_{us}|$, with the potential to be competitive with determinations from kaon decays if experimental errors are reduced, is based on the analysis of sum rules involving inclusive strange hadronic τ decays. A methodology that replaces the traditional use of the OPE on the sum rules by LQCD hadronic-vacuum-polarization functions, which reduces part of the previous systematics, can be found in Ref. [15], together with preliminary results.

References

[1] M. Gonzalez-Alonso, O. Naviliat-Cuncic, and N. Severijns, Prog. Part. Nucl. Phys. **104**, 165 (2019), arXiv:1803.08732 [hep-ph].

[2] Y. Grossman, E. Passemar, and S. Schacht, JHEP **07**, 068 (2020), arXiv:1911.07821 [hep-ph].

[3] A. Bazavov *et al.*, Phys. Rev. D **98**, 074512 (2018), arXiv:1712.09262 [hep-lat].

[4] A. Bazavov *et al.* (Fermilab Lattice, MILC), Phys. Rev. D **99**, 114509 (2019), arXiv:1809.02827 [hep-lat].

[5] S. Aoki *et al.* (Flavour Lattice Averaging Group), (2019), arXiv:1902.08191 [hep-lat].

[6] V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, and J. Portoles, Rev. Mod. Phys. **84**, 399 (2012), arXiv:1107.6001 [hep-ph].

[7] M. Di Carlo, D. Giusti, V. Lubicz, G. Martinelli, C. Sachrajda, F. Sanfilippo, S. Simula, and N. Tantalo, Phys. Rev. D **100**, 034514 (2019), arXiv:1904.08731 [hep-lat].

[8] C. Kane, C. Lehner, S. Meinel, and A. Soni, in *37th International Symposium on Lattice Field Theory* (2019) arXiv:1907.00279 [hep-lat].

[9] A. Desiderio *et al.*, (2020), arXiv:2006.05358 [hep-lat].

[10] M. Moulson (KLEVER Project), J. Phys. Conf. Ser. **1526**, 012028 (2020), arXiv:1912.10037 [hep-ex].

[11] T. Nomura, J. Phys. Conf. Ser. **1526**, 012027 (2020).

[12] N. H. Christ, X. Feng, A. Juttner, A. Lawson, A. Portelli, and C. T. Sachrajda, Phys. Rev. D **94**, 114516 (2016), arXiv:1608.07585 [hep-lat].

[13] N. H. Christ, X. Feng, A. Portelli, and C. T. Sachrajda (RBC, UKQCD), Phys. Rev. D **100**, 114506 (2019), arXiv:1910.10644 [hep-lat].

[14] X. Feng, M. Gorchtein, L.-C. Jin, P.-X. Ma, and C.-Y. Seng, Phys. Rev. Lett. **124**, 192002 (2020), arXiv:2003.09798 [hep-lat].

[15] P. Boyle, R. J. Hudspith, T. Izubuchi, A. Jüttner, C. Lehner, R. Lewis, K. Maltman, H. Ohki, A. Portelli, and M. Spraggs (RBC, UKQCD), Phys. Rev. Lett. **121**, 202003 (2018), arXiv:1803.07228 [hep-lat].