High-precision determination of V_{us} and V_{ud} from lattice QCD

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August 31, 2020

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1 General motivations

The determination of the elements of the quark-mixing Cabibbo-Kobayashi-Maskawa (CKM) matrix provides a stringent test of Standard Model of Particle Physics, which predicts the CKM matrix to be unitary. Finding possible (small) deviations from unitarity or discrepancies between the same CKM matrix element determined from different processes requires high-precision estimates of the individual CKM matrix elements. This letter of interest focuses on determinations of the CKM matrix elements V_{ud} and V_{us}, which describe the transition of an up- to a down- or strange-quark via the weak interaction, respectively and in particular high-precision lattice QCD calculations relevant for extracting V_{ud} and V_{us} from experimentally measured decay rates.

2 $K_{\ell 2}$ decays including radiative corrections

The leptonic decay of a charged kaon $K^\pm \to \ell^\pm \nu_\ell$ ($K_{\ell 2}$ decay) and a charged pion $\pi^\pm \to \ell^\pm \nu_\ell$ ($\pi_{\ell 2}$ decay), provide determinations of the CKM matrix elements V_{us} and V_{ud}, respectively. The lepton in the final state can be either a muon or an electron. Extracting V_{us} (V_{ud}) from these processes requires the experimentally measured decay rate as well as a calculation of an appropriate QCD matrix element, which, if radiative corrections are neglected, is expressed in terms of the kaon (pion) decay constant f_K (f_π). In recent years, lattice calculations of such decay constants have reached a precision of $\lesssim 1\%$ (see [1] for an overview of results). Traditionally, lattice calculations are often done in the isospin symmetric $m_u = m_d$ limit as well as neglecting electromagnetic corrections. These effects are of $O((m_u - m_d)/\Lambda_{\mathrm{QCD}})$ and $O(\alpha)$, respectively, and further increase of precision beyond 1%, requires the inclusion of these effects in the calculation. The formalism to include QED corrections to $\{K,\pi\}_{\ell 2}$ decays in a lattice calculation has recently been worked out in [2]. When including contributions with a virtual photon it is necessary to also add contributions including a final state photon to cancel infrared divergences and the total decay rate $K^\pm \to \ell^\pm \nu_\ell(\gamma)$ is an infrared finite quantity. First lattice calculations of electromatgnetic corrections to $\{K,\pi\}_{\ell 2}$ decays at unphysically heavy quark masses and treating final state photon contributions in the pointlike meson approximation have been published in [3, 4]. In a recent paper [5] final state photon radiation for such processes has been calculated on the lattice.

Our collaboration is currently working on calculating the isospin breaking and QED corrections to $\{K,\pi\}_{\ell 2}$ directly at (close-to-) physical quark masses at a single lattice spacing. A full lattice calculation of all relevant quantities at physical quark masses including a continuum extrapolation could be available with in the time frame of two to three years. This includes non-perturbative renormalisation of the weak Hamiltonian in QCD+QED, isospin breaking effects for sea quarks as well as a lattice determination of real photon emission. The latter will pave the way for a similar calculation in the heavy quark sector. We are also exploring calculating QED corrections in infinite volume analytically to eliminate all power-law suppressed finite volume errors [6].

The inclusion of isospin breaking corrections in the calculation will allow for a more precise determination of V_{us} and V_{ud} from leptonic meson decays in the future. Within the next ten years we aim to increase the overall precision for the necessary lattice calculations to sub-permille accuracy.

3 $K_{\ell 3}$ decays including radiative corrections

The semileptonic kaon decay $K \to \pi \ell \nu_\ell$ ($K_{\ell 3}$ decay) includes a transition of s to a u quark via the weak interaction and therefore provides information on V_{us}. Without electromagnetic corrections the hadronic matrix element relevant for this decay is given by two form factors $f_\pm(q^2)$

$$\langle \pi(p_\pi) | \overline{u}\gamma_\mu s | K(p_K) \rangle = f_+(q^2)(p_K + p_\pi)_\mu + f_-(q^2)(p_K - p_\pi)_\mu \tag{1}$$

with the four-momentum transfer $q^2 = (p_K - p_\pi)^2$. The form factors $f_\pm$ can be calculated *ab initio* using lattice QCD. Similarly to the $\{K,\pi\}_{\ell 2}$ decays discussed in section 2 recent state-of-the-art of lattice calculations have made it necessary to include isospin breaking and QED corrections to further increase the precision. These corrections have previously been estimated in Chiral Perturbation Theory (see, e.g., [7]), however, currently no *ab initio* determination using lattice QCD is available. A first step towards a lattice determination is the development of a formalism how such a calculation can be done. Similar as for the $\{K,\pi\}_{\ell 2}$ in [2], infrared divergences need to be canceled by adding processes including final state photons (see [8] for a detailed discussion). An additional challenge for a lattice calculation of $K_{\ell 3}$ including isospin breaking corrections is the necessity to scan the whole 2D Daliz plot of the kinematically allowed region (whereas $f_{\pm}(q^2)$ in the isospin symmetric limit only depends on a single kinematic variable q^2). A completed formalism, a determination of finite volume corrections and first exploratory lattice calculations could be available within the timeframe of two years. A complete calculation of all relevant contributions at (close-to-) physical quark masses including a continuum extrapolation achieving an overall permille level precision on the lattice calculation for $K_{\ell 3}$ decays will be the long term goal for the next ten years.

4 V_{us} from semileptonic hyperon decays

Complementary information on V_{us} can be obtained from suitable weak baryon decays that include an $s \to u$ transition, in particular semileptonic decays within the baryon octet, e.g. $\Sigma^- \to n\ell^-\overline{\nu}_\ell$, $\Xi^0 \to \Sigma^+\ell^-\overline{\nu}_\ell$, $\Xi^- \to \Lambda\ell^-\overline{\nu}_\ell$ etc. An overview of existing experimental results for these decays can be found in [9]. Besides $\Xi^0 \to \Sigma^+\ell^-\overline{\nu}_\ell$ which has been measured by NA48 [10, 11] no recent measurements exists for any of the other decays, however, LHCb has recently expressed interest [12] to measure semileptonic Hyperon decays. Extracting V_{us} from experimental results for semileptonic Baryon decay $B_2 \to B_1\ell\nu_\ell$ requires knowledge of the respective hadronic matrix element, which is given by six form factors

$$\begin{aligned}\big\langle B_1\big|\overline{u}\,\gamma_\mu(1-\gamma_5)\,s\big|B_2\big\rangle =&\overline{u}_{B_1}(p_{B_1})\Big\{\Big[\gamma_\mu f_1(q^2) - i\frac{\sigma^{\mu\nu}q_\nu}{M_{B_2}+M_{B_1}}\,f_2(q^2) + \frac{q_\mu}{M_{B_2}+M_{B_1}}\,f_3(q^2)\Big]\\ &+\Big[\gamma_\mu g_1(q^2) - i\frac{\sigma^{\mu\nu}q_\nu}{M_{B_2}+M_{B_1}}\,g_2(q^2) + \frac{q_\mu}{M_{B_2}+M_{B_1}}\,g_3(q^2)\Big]\gamma_5\Big\}u_{B_2}(p_{B_2})\end{aligned} \tag{2}$$

with $q = p_{B_2} - p_{B_1}$ and can be calculated in Lattice QCD. Lattice calculations including Baryons are generally more challenging than calculations of similar mesonic processes (like the $K_{\ell 3}$ decay) due to an exponentially growing noise-to-signal ratio and results for the form factors in equation (2) close to the physical point in isospin symmetric QCD would be valuable to provide an alternative determination of V_{us} in conjunction with experimental data. We are planing to do such a calculation and first results at a single lattice spacing could be available with in the next two years with a full physical point calculation including continuum extrapolation aiming at 1% overall precision for the relevant form factors in the years to follow. Whether or not a further increase in precision will be required in the long term will depend on future experimental efforts.

References

[1] S. Aoki *et al.* (Flavour Lattice Averaging Group), Eur. Phys. J. C **80**, 113 (2020), arXiv:1902.08191 [hep-lat] .

[2] N. Carrasco, V. Lubicz, G. Martinelli, C. T. Sachrajda, N. Tantalo, C. Tarantino, and M. Testa, Phys. Rev. **D91**, 074506 (2015), arXiv:1502.00257 [hep-lat] .

[3] D. Giusti, V. Lubicz, G. Martinelli, C. T. Sachrajda, F. Sanfilippo, S. Simula, N. Tantalo, and C. Tarantino, Phys. Rev. Lett. **120**, 072001 (2018), arXiv:1711.06537 [hep-lat] .

[4] M. Di Carlo, D. Giusti, V. Lubicz, G. Martinelli, C. Sachrajda, F. Sanfilippo, S. Simula, and N. Tantalo, Phys. Rev. D **100**, 034514 (2019), arXiv:1904.08731 [hep-lat] .

[5] A. Desiderio *et al.*, (2020), arXiv:2006.05358 [hep-lat] .

[6] X. Feng and L. Jin, Phys. Rev. D **100**, 094509 (2019), arXiv:1812.09817 [hep-lat] .

[7] V. Cirigliano, M. Giannotti, and H. Neufeld, JHEP **11**, 006 (2008), arXiv:0807.4507 [hep-ph] .

[8] C. Sachrajda, M. Di Carlo, G. Martinelli, D. Giusti, V. Lubicz, F. Sanfilippo, S. Simula, and N. Tantalo, in *PoS LATTICE2019* (2019) arXiv:1910.07342 [hep-lat] .

[9] M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. **D98**, 030001 (2018).

[10] J. Batley *et al.* (NA48/I), Phys. Lett. B **645**, 36 (2007), arXiv:hep-ex/0612043 .

[11] J. Batley *et al.* (NA48/1), Phys. Lett. B **720**, 105 (2013), arXiv:1212.3131 [hep-ex] .

[12] A. B. Rodríguez, Journal of Physics: Conference Series **1526**, 012022 (2020).