# Rare strange-to-down processes from lattice QCD

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## 1 General motivations

In the Standard Model (SM) a strange quark cannot decay into a down quark at tree-level. However such transition is possible through radiative corrections involving a second-order weak decay $s \to u, c, t \to d$. Because of that, hadronic processes requiring $s \to d$ transitions are expected to be extremely suppressed in the SM. Such processes are therefore prime targets to constrain experimentally extensions of the SM featuring tree-level flavour-changing neutral currents (FCNC).

Rare $s \to d$ processes featuring light hadrons often receive significant contribution from long-distance hadronic effects which cannot be described in perturbative QCD. In order to search for new physics using these decays, it is therefore crucial to produce SM predictions from precise and reliable non-perturbative QCD calculations and we believe that lattice simulations are uniquely positioned to provide such predictions. Therefore in this Letter of Interest we propose a roadmap of future lattice QCD calculations designed to assist current experimental efforts to find new physics through rare $s \to d$ decays.

## 2 Rare kaon decays $K \to \pi\nu\bar{\nu}$

The rare kaon decays $K \to \pi\nu\bar{\nu}$ have attracted increasing interest during the past few decades. As flavor changing neutral current processes, these decays are highly suppressed in the standard model (SM) and thus provide ideal probes for the observation of new physics effects. The known branching ratio measurement [1] of $K^+ \to \pi^+\nu\bar{\nu}$ is almost twice the SM prediction [2], but with a 60-70% uncertainty it is still consistent with SM. The current experiment, NA62 at CERN, which aims at an observation of $\mathcal{O}(100)$ events and a 10%-precision measurement of $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$, recently reports an upper limit of $1.78 \times 10^{-10}$ for the $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ at 90% CL [3].

Theoretically these decays are known to be short-distance dominated and theoretically clean. The long-distance contributions are safely neglected in $K_L \to \pi^0\nu\bar{\nu}$ and are expected to be small, perhaps of $O(5\text{-}10\%)$, in $K^+ \to \pi^+\nu\bar{\nu}$ decays. Though small, by including the long-distance contribution estimated from chiral perturbation theory, the branching ratio $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ is enhanced by 6% [4], which is comparable to the 8% total Standard Model error [2]. Since it will be possible to compare the SM

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predictions with the new experimental measurement of $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$ relatively soon, a lattice QCD calculation of the long-distance contributions is important and timely.

In the past years, we have developed the theoretical framework [5] and performed exploratory numerical calculations with the pion mass approaching its physical value [6, 7, 8]. To reach the full physical kinematics, we also need a fine lattice to avoid lattice artifacts from the physical mass of the charm quark. We are currently carrying out a physical-kinematics calculation of the long-distance contribution to $K \to \pi\nu\bar{\nu}$ with a target of 30% total uncertainties. A more accurate future calculation will be possible but require substantial computer resources.

## 3 Rare kaon decays $K \to \pi\ell^+\ell^-$

These decays are expected to be extensively observed in the next years through the NA62, and LHCb experiments. They are dominated by SM long-distance effects featuring an electroproduction of the lepton pair through the intermediate process $K \to \pi\gamma^*$. Using Ward-Takahashi identities, the amplitude of this decay can generally be written

$$\mathcal{A}_\mu[K(k) \to \pi(p)\gamma^*(q)] = -i\frac{G_F}{(4\pi)^2}[q^2(k+p)_\mu - (M_K^2 - M_\pi^2)q_\mu]V(z)\,, \tag{1}$$

with $z = q^2/M_K^2$. A reliable SM model prediction of these decay requires a precise description of the form factor $V(z)$ dependence on $z$. A parametrisation commonly accepted [9] to describe well the experimental data is

$$V(z) = a_c + b_c z + V_{\pi\pi}(z)\,, \tag{2}$$

where $c \in \{+, S\}$ indicates the kaon charge, $a_c$ and $b_c$ are two unknown real constants, and $V_{\pi\pi}(z)$ describes the additional contributions above the $q^2 = 4M_\pi^2$ threshold. The constants $a_c$ and $b_c$ are currently only known from experimental measurement [10] or phenomenological descriptions [11]. These constants can be correlated to other rare decays, for example in the $B$ sector, through lepton flavour universality violating extensions of the SM [12].

Our collaboration pioneered the first calculations of this rare kaon decay in an unphysical context [13, 14]. Additionally we have been actively working over the past years on designing simulation setup directly at the physical point [15]. We believe that over the next 5-10 years, lattice QCD will be in a position to produce predictions of $a_S, a_+, b_S$, and $b_+$ with uncertainties below the 10% level.

## 4 Rare kaon decays $K_L \to \mu^+\mu^-$

The well-measured rare decays $K_L \to \mu^+\mu^-$ suffers from a dominant long-distance SM contribution. Lattice QCD has the potential to enable the search of new physics though these decays. Future plans regarding this process are described in a companion letter of interest.

## 5 Rare hyperon decays $\Sigma^+ \to p\ell^+\ell^-$

There are growing efforts in the LHCb experiment [16] to analyse these decays. The rare decay $\Sigma^+ \to p\ell^+\ell^-$ can be seen as a baryonic version of the rare kaon decay $K^+ \to \pi^+\ell^+\ell^-$ discussed in Sec. 3. Similarly, it is dominated by long-distance effects through the intermediate process $\Sigma^+ \to p\gamma^*$. Where rare kaon decays $K^+ \to \pi^+\gamma^*$ only need one form factor to describe the amplitude, the decay $\Sigma^+ \to p\gamma^*$ needs four of them because of the higher spin of the external states. The extensive work from [17] used various phenomenological approaches to constrain these form factors. However the results obtained give a fairly wide range of values for the branching ratio of the decay.

We are currently preparing a manuscript presenting a theoretical framework for a lattice prediction of the $\Sigma^+ \to p\gamma^*$ amplitude, which can be seen as a generalisation of our past work on rare kaon decays [13]. We hope to achieve an unphysical proof-of-concept calculation within the next 2 years. Baryonic lattice correlation functions are notorious for the exponential degradation of their signal-to-noise ratio with the time separation of the baryon operators. Therefore such calculation is expected to be extremely challenging from a numerical point of view and will probably require a breakthrough in methods to build lattice correlators. A possibly promising direction is the recent rapid development of multi-level algorithms [18, 19]. For the long term future, we are aiming at providing at least the $q^2 = 0$ of the form factors at the physical point with uncertainties below the 10% level.

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