

Discovering new physics in rare kaon decays

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The increasing precision and expanding reach of lattice QCD calculations create new opportunities to search for phenomena that lie outside of the standard model. With more accurate standard model predictions, previous experimental results acquire greater sensitivity to beyond-the-standard-model phenomena at higher energies and with smaller couplings. These new theoretical predictions may also motivate future experiments if the results from these experiments can be predicted with greater accuracy from the standard model. In this letter of interest we focus on low-energy phenomena involving kaon mixing and decay, both first- and second-order weak processes. This is the region in which current lattice calculations are most accurate with well-understood systematic errors, where the barriers to obtaining predictions with substantially smaller errors appear surmountable. Examples of phenomena where exciting theoretical improvements should be possible in the next decade include:

1. Direct CP violation and ε' . The parameter ε' , a measure of direct CP violation in $K \rightarrow \pi\pi$ decay, has been computed in the standard model with 40% error [1]. The largest part of this error is systematic and can be reduced by carrying out larger scale and more complex lattice calculations. This calculation must be repeated at two or three lattice spacings so that a continuum limit can be taken. The effects of an active charm quark should be included so that the GIM cancellation can be realized without neglecting power-law corrections falling as an inverse power of the charm quark mass. Especially challenging is calculation of isospin-breaking corrections. While normally of order α_{EM} , these are $\Delta I = 1/2$ -rule enhanced by a factor of twenty for this process and represent one of the largest current systematic errors. Here new methods must be developed if a future lattice calculation is to verify and refine the current ChPT estimates [2], although some first small steps have already been developed [3].

2. Indirect CP violation and ε . Important lattice QCD calculations of the kaon bag parameter B_K led to early predictions for the short-distance part of ε which would now be accurate on the one-percent level, except for the larger uncertainties arising from the current errors for V_{cb} – errors which should substantially decrease over the next decade.

However, to reach sub-percent accuracy, the few-percent long-distance correction to ε must be computed. This is an explicit second-order-weak process requiring the $K^0 - \bar{K}^0$ matrix element of a sum of bilocal operators which has already been explored in lattice QCD [4] and will be undertaken for physical quark masses during the coming year. Here an active charm quark and lattice realization of GIM cancellation is critical. Thus, very computationally expensive calculations with choices of the inverse lattice spacing which vary between 3 and 5 GeV are required to obtain a continuum limit and 10% accuracy for this potentially 5% effect.

3. The $K_L - K_S$ mass difference. With a measured value of $3.482(6) \times 10^{-12}$ MeV this small mass difference has sensitivity to physics at the 1,000 TeV scale and yet has been predicted by the standard model with only 36% accuracy [5]. Exploratory calculations [6] have shown that this quantity can be computed with controlled errors using lattice QCD and a large-scale calculation with physical quark masses is nearing completion. While reaching the experimental accuracy for this important quantity is likely not possible within the coming decade, errors on the five-percent level may be possible. A natural companion to item **2.** above, accurate results for this mass difference require a sequence of gauge ensembles with small, decreasing lattice spacings and high-statistics evaluations on possibly thousands of configurations.

4. $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay. This important decay is being studied by NA62 and has a few-percent long-distance component which can be computed using lattice QCD with methods similar to items **2.** and **3.** above. It is discussed in a companion [LoI](#).

5. $K_L \rightarrow \mu^+ \mu^-$ decay. This accurately-measured strangeness-changing neutral current process does not allow a corresponding test of the standard model at second-order because of the background resulting from a two-photon, $O(\alpha_{EM}^2 G_F)$ $\mu^+ \mu^-$ decay amplitude. However, this background amplitude may well be possible to compute using the methods of lattice QCD [7]. The first step in computing this amplitude has been taken by accurately computing the less challenging $O(\alpha_{EM}^2)$ process $\pi^0 \rightarrow e^+ e^-$ [8]. A second step is now underway to compute the $K_L \rightarrow \gamma \gamma$ decay amplitude. The final step of a 10% calculation of the two-intermediate-photon contribution to $K_L \rightarrow \mu^+ \mu^-$ may be possible within the next five years but some theoretical advances are needed to control the finite-volume effects in this decay where there are many-particle intermediate states with energy less than that of the decaying kaon.

These are all very challenging lattice calculations which require substantial access to the most powerful HPC resources and the continuing large investment in software and algorithm development needed to make effective use of cutting edge hardware.

In light of the breathtaking advances in theoretical technique and experimental capability, such high-precision study of rare processes may be ripe for new experiments that exploit our increasing understanding and control of QCD. The strong interactions among the quarks create unique opportunities for precision experiments but often lead to difficulties in making equally precise predictions – difficulties which in many cases are now being overcome by the methods of lattice QCD.

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