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The quest for explaining the top-row CKM unitarity deficit

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Abstract: Symmetries of the weak sector of the Standard Model and its completeness find an exact mathematical realization in the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. Of various relations among its elements, the top-row unitarity relation is by far the one known with the highest precision. The last few years have seen a rapid development in both the theory and experiments related to the extraction of the top-row CKM matrix elements and the respective unitarity relation, as quoted in the 2020 PDG [1]:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(3)_{V_{ud}}(4)_{V_{us}}. \tag{1}$$

The apparent 3σ deviation from unitarity points towards the possibility of BSM physics. Therefore, it is important to further reduce all the SM uncertainties in both V_{ud} and V_{us} in order to reach a level sufficient to claim a discovery.

V_{ud} from nuclear and neutron beta decays: The most precise extraction of V_{ud} is warranted by the superallowed $0^+ - 0^+$ nuclear decays with the central formula

$$|V_{ud}^{0^+-0^+}|^2 = 2984.43\,\mathrm{s}/[\mathcal{F}t(1+\Delta_R^V)] \quad \text{with} \quad |V_{ud}^{0^+-0^+}| = 0.9737(1)_{\mathcal{F}t}(1)_{\mathrm{RC}}. \tag{2}$$

Above, Δ_R^V is the universal, free-neutron radiative correction (RC), while the universal, decay-independent modified statistical rate function $\mathcal{F}t = ft(1+\delta'_{\mathrm{R}})(1+\delta_{\mathrm{NS}}-\delta_{\mathrm{C}})$ is obtained from the statistical rate function ft measured for each decay by absorbing the decay-specific corrections that are explained below. Similarly, free neutron decay gives

$$|V_{ud}^{\mathrm{free\,n}}|^2 = 5024.7\,\mathrm{s}/[\tau_n(1+3g_A^2)(1+\Delta_R)] \quad \text{with} \quad |V_{ud}^{\mathrm{free\,n}}| = 0.9733(3)_{\tau_n}(3)_{g_A}(1)_{\mathrm{RC}}, \tag{3}$$

with τ_n the neutron lifetime averaged over the UCN experiments (extra uncertainty exists if one includes the beam-bottle discrepancy), g_A the nucleon axial coupling, and Δ_R the RC differing from Δ_R^V by absorbing the neutron decay-specific QED correction. While the precision of g_A has recently been improved by a factor of 4 [2], plans to further reduce experimental uncertainties exist [3] to match the precision of the superallowed decays.

The main single source of theoretical uncertainty, the γW box contribution residing in Δ_R^V, has recently been re-evaluated with dispersion relations (DR). This led to halving the respective uncertainty but shifted its central value significantly [4, 5], a result confirmed by two subsequent studies [6, 7]. Meanwhile, remaining sources of model dependence reside in $\mathcal{F}t$: δ_{NS} accounts for the nuclear modification of the free-neutron RC Δ_R^V; δ_{C} arises from isospin symmetry breaking (ISB) effects in the nuclear states. At present, the standard way of computing both corrections is within the nuclear shell model [8]. The situation is however far from settled as the comparison amongst different nuclear models does not generally support current uncertainty estimates [9–12], and the validity of the formalism of Ref. [8] has been questioned [5, 13–15]. Modern ab-initio calculations with controlled uncertainties are needed, and so are ways to relate these calculations to experimental data for model-independent uncertainties. Our proposal entails:

- **Direct lattice QCD calculation of the axial γW box correction to neutron decay** will follow the analogous calculation on the pion [16], and will provide a further independent check of Δ_R^V.
- **Improved experimental input in the DR calculation of the axial γW box**: the analysis of Refs. [4, 5] relies on older data on the inclusive structure function $F_3^{\nu p+\bar{\nu}p}$ from bubble chamber experiments, plagued by large uncertainties. An additional experimental program on $\nu/\bar{\nu}$ scattering on H/D target at the near detector at DUNE [17] and T2HK will provide a better quality model-independent input.
- **Modern ab-initio calculations of nuclear corrections** δ_{C}, δ_{NS} to the rates of superallowed nuclear transitions will dramatically improve the robustness of the respective theoretical uncertainties. A combination of different methods, e.g. Quantum Monte Carlo, Lorentz Integral Transform and Coupled Cluster may be suitable choices depending on the particular transition ranging from $^{10}C \to {}^{10}B$ to $^{74}Rb \to {}^{74}Kr$.

- **Charge radii and neutron skins across the superallowed isomultiplet as a measure of ISB effects**: $\delta_{\rm C}$ arises from ISB effects that generate a mismatch in the radial proton and neutron functions in the parent and daughter nucleus, respectively. The same effects generate neutron skins and change the nuclear charge radius across the isomultiplet. We propose a theoretical study that will relate these observables to $\delta_{\rm C}$ in a model-independent way. Based on this, a series of measurements constraining the proton and neutron distributions in several superallowed isomultiplets that control the fit of the $\mathcal{F}t$ will be proposed. Charge radii of stable daughter nuclei are known to few parts in 10^4 [18]. Parity-violating electron scattering at Jefferson Lab and at the future facility MESA at Mainz will allow to extract neutron radii of selected stable superallowed daughters to few per mille [19]. Charge radii of unstable parent nuclei can be measured at rare-isotope facilities, e.g. ISOLDE and FRIB, via laser spectroscopy or reaction/interaction cross section measurements.

- **Electroweak RC to the Gamow-Teller rates within the dispersive formalism** is relevant for comparing g_A measured in neutron decay asymmetry experiments to lattice QCD calculations. Input from ab-initio methods will allow to address nuclear effects important for the extraction of V_{ud} from nuclear mirror decays. This formalism is directly applicable to neutral current processes, e.g., to evaluate the hyperfine contribution to the nucleon and nuclear anapole moments, of interest in parity violation in electron scattering and atomic systems.

V_{us} **from kaon decays**: The largest uncertainty in the quoted value of $|V_{us}| = 0.2245(8)$ in PDG 2020 [1] comes from the discrepancy between its extraction from $K_{\mu 2}$ and K_{l3}. The results with $N_f = 2+1+1$ lattice input read:

$$|V_{us}^{K_{\mu 2}}| = 0.2252(5), \quad |V_{us}^{K_{l3}}| = 0.2231(4)_{\rm exp+RC}(6)_{\rm lattice}. \tag{4}$$

The disagreement of $\sim 3\sigma$ indicates, apart from possible BSM explanations, the existence of yet unidentified SM systematic errors not reflected in the existing error estimation. Therefore it is necessary to revisit all the SM inputs thoroughly. The $K_{\mu 2}$ result comes from the ratio $R_A = \Gamma_{K_{\mu 2}}/\Gamma_{\pi_{\mu 2}}$ [20] which is independent of the uncertainties from the short-distance EM effects as well as the unknown electromagnetic LECs [21], and is believed to be more robust, given the recent lattice QCD analysis [22]. Meanwhile, $|V_{us}|$ is extracted from K_{l3} through the master formula [23]:

$$\Gamma_{K_{l3}} = \frac{C_K^2 G_F^2 M_K^5}{128\pi^3} S_{\rm EW} |V_{us}|^2 |f_+^{\pi^- K^0}(0)|^2 I_{Kl}^{(0)}(\lambda_i)(1 + \delta_{\rm em}^{Kl} + \delta_{\rm SU(2)}^{K\pi}). \tag{5}$$

A common belief is that the $K\pi$ form factor $f_+^{K^0\pi^-}(0)$ is the main culprit of the discrepancy. To clarify this issue the lattice community will aim to reconcile the conflicting numbers of Refs. [24] and [25]. Of the remaining corrections, both $I_{Kl}^{(0)}$ and $\delta_{\rm SU(2)}^{K\pi}$ are treated using ChPT + dispersion relation relying on the experimental input, and hence are more under control. This leaves $\delta_{\rm em}^{Kl}$, the long-distance EM correction, as the main source of model-dependence and should be carefully reanalyzed. In the existing literature $\delta_{\rm em}^{Kl}$ is calculated within the framework of ChPT to $\mathcal{O}(e^2p^2)$ [26], and its uncertainties are two-fold: (1) (0.11-0.16)% from the unknown electromagnetic LEC at $\mathcal{O}(e^2p^2)$ depending on the decay channel, and (2) a universal uncertainty of 0.19% from $\mathcal{O}(e^2p^4)$ effects. Here we propose several future steps to reduce the uncertainties from the above two sources, as well as other possible systematic effects:

- **Lattice QCD calculations of all the needed LECs at** $\mathcal{O}(e^2p^2)$ [27]. Calculating the γW box diagram in the SU(3) limit will fix LECs $\{X_i\}$, whereas that of four-point correlation functions consisting of time-ordered product of vector and axial currents will fix LECs $\{K_i\}$.

- **Evaluation of the** $\mathcal{O}(e^2p^4)$ **effects**: one can opt for a two-loop calculation in ChPT, which requires the knowledge of $\mathcal{O}(e^2p^2)$ LECs (which adds to the motivation of the lattice calculations proposed above). An alternative approach is to make use of the new ChPT + current algebra formalism [28] that automatically resums a subset of EM corrections to all orders in the chiral expansion. These resummed contributions will be treated using dispersion relations, with a possible input from lattice QCD.

Another strategy proposes to extract $|V_{us}/V_{ud}|$ via the ratio $R_V = \Gamma_{K_{l3}}/\Gamma_{\pi_{l3}}$ [29]. However, unlike R_A, this ratio is sensitive to electromagnetic LECs at $\mathcal{O}(e^2p^2)$, so our proposed calculations above will also help improving the SM theory prediction of R_V, which may shed new light on the $K_{\mu 2} - K_{l3}$ discrepancy.

Summary: This proposal addresses all theoretical uncertainties in V_{ud} and V_{us} via a combination of novel calculations in lattice QCD and ab-initio nuclear theory, incorporating the chiral and perturbative QCD limits, with the dispersion theory connecting all of them together. We envision a vibrant future experimental program at neutrino and electron scattering facilities and at rare isotope sources, which will amount to a model-independent way to estimate uncertainties and will directly impact the search for BSM physics with the test of CKM unitarity in its top row.

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