Snowmass2021 - Letter of Interest

Using lattice QCD for the hadronic contributions to the muon $g-2$

Snowmass Topical Groups:

■ (RF3) Small Experiments
■ (TF05) Lattice gauge theory
■ (EF05) QCD and Strong Interactions: Precision QCD
■ (CompF2) Theoretical calculations and simulation

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The anomalous magnetic moment of the muon is currently measured to a precision of around 0.5 ppm [1] and the theoretical uncertainty on its Standard Model prediction [2] is commensurate with the current experimental status. New experiments at Fermilab (which started running in 2018) and at J-PARC plan to reduce the measurement errors by about a factor of four [3, 4]. The goal of these efforts is to probe the observed difference of 3.7σ between Standard-Model theory and experiment, a persistent hint for physics beyond the Standard Model. However, without improvements on the theoretical side, the discovery potential of these efforts may be limited.

The current theory error on a_μ is dominated by the uncertainties in the hadronic contributions. The two leading hadronic contributions are the $O(\alpha_{\rm EM}^2)$ hadronic vacuum polarization (HVP) term and the $O(\alpha_{\rm EM}^3)$ hadronic light-by-light (HLbL) term. In a joint Fermilab Lattice/HPQCD/MILC Collaborations project, we are targeting the larger HVP term.

The HVP contribution can be obtained from experimental data for $e^+e^- \to$ hadrons and dispersive methods, see for example Refs. [5, 6]. The current precision in HVP from these data driven methods is about 0.4%, with, however, some scatter between the results from different groups. There are also some tensions between different measurements of electron-positron cross section into the two-pion final state, which dominates the dispersive integral [7]. Lattice-QCD calculations of $a_\mu^{\rm HVP}$ do not suffer from these experimental complications, and their sources of systematic error are independent. A lattice-QCD calculation of $a_\mu^{\rm HVP}$ with sub-percent level uncertainty would provide a valuable test of the data-driven results. Current lattice results are at about 2% uncertainty levels, except for a very recent one with sub-per-cent precision [8] albeit in tension with the dispersive results.

After a first study of the strong isospin breaking corrections [9], we completed a lattice-QCD calculation of the leading-order HVP contribution [10] on the MILC Collaboration's (2+1+1)-flavor HISQ ensembles last year. Follow-up work is ongoing and is targeting the dominant uncertainties in our current result to improve our calculation to sub-percent level precision, in time to be of use to the Muon $g-2$ Experiment. Significant sources of uncertainty are due to statistics, the lattice scale, finite volume, and the subleading disconnected and isospin breaking corrections, all of which must be reduced in order to reach our precision goal. Each of these are being considered in separate projects. We note that a lattice calculation of the hadronic vacuum polarization function can provide information on related quantities, including the electron $g-2$, the hadronic contributions to the running of α and the weak mixing angle [11], among others.

Our calculation of the disconnected and strong isospin breaking corrections is in progress [12], and we expect to have final results this coming year. Another urgent task is the calculation of the QED corrections, which are intertwined with the strong isospin breaking correction. We plan to analyze a dynamical QED+QCD ensemble, but we are also developing a new fermion-determinant reweighting method for the lowest-order corrections. Building up infrastructure for QED calculations will be beneficial to other quantities. It would be interesting to use the QED infrastructure for a lattice calculation of the hadronic light-by-light contribution. A natural application is adding isospin corrections, including structure-dependent radiative effects, to our quark-flavor physics calculations. However, further theoretical work is needed to adapt and develop methods that are applicable to B-meson quantities.

An interesting method to tame the growing statistical errors in the vector-current correlator at large Euclidean times is to supplement the two-point correlator with additional correlation functions of two-pion operators, which are then used in a spectral analysis to reconstruct the large t tail of the vector current correlator [13, 14]. This project lays the groundwork for adding a completely new set of observables to lattice QCD studies with staggered fermions. Here, we expect that the large library of HISQ ensembles, which has proven to be a powerful tool for precision quark-flavor physics, will enable meaningful studies of these more challenging problems. Possible extensions include weak decays to resonances (such as the $B \to K^*\ell\ell$ transition) or two-hadron final states. Further extensions enabled by studies of two-current matrix elements as discussed in our companion LOI on inclusive B-decays may include two-pion systems as a test bed to explore the feasibility of nucleon-pion and two-nucleon calculations on the HISQ ensembles.

Finally, it is worth noting that several of us are members of the Muon g-2 Theory Initiative, which very recently produced a white paper reviewing the status of Standard Model theory for the muon $g-2$ [2]. This work will continue, certainly for the duration of the experimental programs at Fermilab and J-PARC. The focused workshops organized by the Initiative have enabled discussions and collaboration between theorists using different methods, which has provided more detailed cross checks and comparisons of the different methods. The Initiative plans to publish updated SM predictions ahead of each new major experimental update.

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