Calculations of nucleon electric dipole moments on a lattice with chiral fermions

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1 Introduction

This Letter of Interest discusses the need to study contributions of various effective CP-violating ($\cancel{CP}$) quark-gluon interactions to the nucleon Electric Dipole Moments (nEDMs) from first principles using lattice QCD calculations. Limits from measurements of particle EDMs provide some of the tightest constraints on extensions of the Standard Model such as SUSY. Nucleon EDMs are of particular interest as they additionally offer insight into baryogenesis and the strong CP problem.

While first detection of EDM in a nucleon or a nucleus would be a colossal breakthrough, understanding the nature of underlying CP violation mechanism will require measurements of EDMs in multiple systems such as proton, neutron, nuclei, and molecules. Precise ab initio knowledge of how various kinds of effective quark-gluon $\cancel{CP}$ interactions contribute to "intrinsic" EDMs of protons and neutrons will be necessary for this program.

2 CP violation and EDMs of nucleons/light nuclei

CP violation is one of the necessary conditions for existence of baryon matter, as concluded by Sakharov [1]. Although there is $\cancel{CP}$ interaction within the Standard Model, it contributes to nEDM only $\sim 10^{-31}e$ cm, which is negligible compared to the current experimental limit for the neutron $|d_n| < 1.5 \div 2.9 \cdot 10^{-26}e$ cm [2, 3]. Upcoming neutron EDM experiments at ILL, SNS, PSI, and TRIUMF are projected to improve the sensitivity by two orders of magnitude and constrain the nEDM to $10^{-28}e$ cm. Proposed experiments to measure EDMs of protons and light nuclei in storage rings are especially exciting as they have potential to reach precision $\sim 10^{-29}e$ cm [4, 5] and, taken together, discriminate between different kinds of $\cancel{CP}$ interactions. Prototype storage-ring proton EDM experiment may start construction after 2022, followed by precision experiment after 2027 [6, 7, 8]. These experiments will then be extended to measure EDMs of ^{2}D, ^{3}H and ^{3}He.

Measurements of EDMs in light nuclei will play an especially important role in revealing the nature of CP violation. Combined together with measurements of *intrinsic* EDMs of the proton and the neutron, they may allow us to separate $\cancel{CP}$ due to the QCD $\bar\theta$-term and $\cancel{CP}$ extensions of the Standard Model [9, 10, 11, 12]. This requries knowing their contributions to nucleon intrinsic EDM and $\cancel{CP}$ nucleon interactions from first principles. In the light nuclei, contributions of $\cancel{CP}$ nucleon interactions to EDMs have been computed in chiral perturbation theory [13]. However, contributions to the proton and neutron intrinsic EDMs

$$d_{n,p} = \sum_q g^q_{Tn,p} d_q + d^{\bar\theta}_{n,p}\bar\theta_{\mathrm{QCD}} + d^{\delta_W}_{n,p}\delta_W + \sum_q d^{\delta_q}\delta_q + \dots \qquad (1)$$

can be reliably computed only using lattice QCD. So far, only contributions of quark EDMs ($g^q_{Tn,p}$) have been computed on a lattice with certainty [14]. However, contributions of other (effective) $\cancel{CP}$ quark-gluon interactions such as $\bar\theta_{\mathrm{QCD}}$-term ($d^{\bar\theta}$), Weinberg 3-gluon term (d^{δ_W}), quark chromo-EDMs (d^{δ_q}), remain unknown.

3 Nucleon EDMs from lattice QCD

Studies of nucleon structure using lattice QCD have progressed dramatically in the recent years [15], including the recent calculation of the nucleon axial charge with sub-percent precision [16]. Computing nucleon vector and

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axial form factors and charge radius with full control of systematic errors is underway. However, determination of the nucleon EDM, a P, T-odd matrix element induced by $\not{C}\not{P}$ perturbation to QCD action, remains challenging. Apart from the contributions of quark EDMs [14], EDMs induced by other various $\not{C}\not{P}$ interactions require sophisticated methods. A concerted lattice QCD program is necessary to ensure that theoretical input is available in time to analyze the future precise results for proton, deuteron, ^{3}H, and ^{3}He.

3.1 $\bar\theta_{\mathrm{QCD}}$ term-induced nEDM

Constraining the $\bar\theta_{\mathrm{QCD}}$-term is one of the main outcomes of neutron EDM measurements. Most of the lattice results for $d^{\bar\theta}$ published prior to 2016 were overestimated due to mixing with nucleon magnetic moment in CP-broken QCD vacuum, and with proper correction became compatible with zero [17]. A particular challenge in computing $d^{\bar\theta}$ is the global nature of the integer-valued topological charge $Q = \frac{1}{32\pi^2}\int d^4x(G^a_{\mu\nu}\cdot\tilde{G}^{a\mu\nu})$. Reproducing this feature on a lattice required "cooling" or, more recently, "gradient flow" methods [18]. Recent calculations with the latter are promising [19, 20], albeit performed away from the physical point.

Rigorous calculation of $\bar\theta_{\mathrm{QCD}}$-induced nEDM using traditional methods is challenging. First, statistical noise due to the global θ-term will inevitably become a problem in larger spatial volume. Second, the effect of the $\bar\theta_{\mathrm{QCD}}$-term should scale as $d^{\bar\theta} \propto m_{u,d}$ at small quark masses, which may be violated if chiral symmetry is broken by lattice discretization of fermion fields. Both these problems may be overcome by using chirally symmetric lattice fermions in uniform background electric field, in which $d^{\bar\theta}$ can be rigorously related to local topological charge density [21]. Additionally, sampling of the nucleon correlation with lumps of topological charge (instantons) may be improved by exploiting correlation of the fermion (near-)zero modes with topology.

3.2 Dimension-6 $\not{C}\not{P}$ interactions

CP-violating extensions of the Standard model contribute to nucleon EDMs through effective quark-gluon operators of higher mass dimension, such as quark chromo-EDMs, 3-gluon (Weinberg), and 4-quark operators. Calculations with quark chromo-EDMs with physical quark masses have demonstrated that good statistical precision is within reach of modern calculations [22, 23]. These quantities have to be renormalized and converted from lattice to the $\overline{MS}$ scheme, including subtraction of mixing with lower-dimension operators due to the lattice cutoff scale $\Lambda_{UV} = a^{-1}$. Perturbative conversion factors for the chromo-EDM [24] and the 3-gluon $\not{C}\not{P}$ operator [25] have been computed for the traditional momentum-subtraction scheme on a lattice (RI-MOM). While these methods are straightforward in principle, practical implementation may be difficult due to the mixing of large number of noisy lattice operators, including those appearing due to required (Landau-)gauge fixing. Also, chirally symmetric QCD action is highly preferable for such nEDM calculations in order to eliminate $O(a)$ discretization effects, e.g., the "clover term" that would otherwise mix with chromo-EDM due to parity breaking.

Analyzing the 3-gluon $\not{C}\not{P}$ interaction is more challenging due to the inherent statistical noise of pure-gluon operators. So far, attempts to calculate its contribution to nEDM have not produced statistically significant results, at least with quark masses towards the physical point [26]. Furthermore, renormalization of the 3-gluon operator will be even more challenging than quark chromo-EDM due statistical fluctuations and the larger number of operators mixing due to fixed gauge [25]. Alternative renormalization schemes may have to be considered, e.g., involving combination of gradient flow and perturbative matching [27].

Contributions of $\not{C}\not{P}$ 4-quark operators have not been studied yet. Evaluating their full contribution will be a challenging task for lattice QCD due to associated so-called quark-disconnected loops. However, several recently developed methodologies are expected to alleviate these numerical problems [28, 29].

4 Proposed study

We propose calculations of proton- and neutron-EDMs induced by the $\bar\theta_{\mathrm{QCD}}$-term, 3-gluon, quark chromo-EDM and 4-quark $\not{C}\not{P}$ interactions in QCD with physical chiral-symmetric quarks. The volume of the lattice $\approx (5.5\ \mathrm{fm})^3$ should be sufficient to eliminate all but negligible finite-volume effects. The lattice spacing $a = 0.082\ldots0.114$ fm combined with automatic $O(a)$-improvement of the fermion action due to the chiral symmetry will be sufficient for reliable continuum extrapolation. Based on previous calculations, a rough estimate of the resources required is $(0.5\ldots1)\cdot10^9$ core-hours.

References

[1] A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. **5**, 32 (1967), [JETP Lett.5,24(1967); Sov. Phys. Usp.34,no.5,392(1991); Usp. Fiz. Nauk161,no.5,61(1991)].

[2] B. Graner, Y. Chen, E. G. Lindahl, and B. R. Heckel, Phys. Rev. Lett. **116**, 161601 (2016), [Erratum: Phys. Rev. Lett.119,no.11,119901(2017)], arXiv:1601.04339 [physics.atom-ph] .

[3] C. A. Baker *et al.*, Phys. Rev. Lett. **97**, 131801 (2006), arXiv:hep-ex/0602020 [hep-ex] .

[4] F. J. M. Farley, K. Jungmann, J. P. Miller, W. M. Morse, Y. F. Orlov, B. L. Roberts, Y. K. Semertzidis, A. Silenko, and E. J. Stephenson, Phys. Rev. Lett. **93**, 052001 (2004), arXiv:hep-ex/0307006 [hep-ex] .

[5] J. Pretz (JEDI), *Proceedings, 5th International Symposium on Symmetries in Subatomic Physics (SSP 2012): Groningen, The Netherlands, June 18-22, 2012*, Hyperfine Interact. **214**, 111 (2013), arXiv:1301.2937 [hep-ex] .

[6] F. Abusaif *et al.*, (2019), arXiv:1912.07881 [hep-ex] .

[7] C. Carli, P. Fierlinger, P. Lenisa, J. Pretz, F. Rathmann, E. Stephenson, and H. Ströher, "Storage rings for the search of charged-particle electric dipole moments," SnowMass-21 Letter of Interest (2020).

[8] N. N. Nikolaev, F. Rathmann, R. Talman, H. Ströher, *et al.*, "Test of the standard model and search for physics beyond," SnowMass-21 Letter of Interest (2020).

[9] J. de Vries, R. Higa, C. P. Liu, E. Mereghetti, I. Stetcu, R. G. E. Timmermans, and U. van Kolck, Phys. Rev. **C84**, 065501 (2011), arXiv:1109.3604 [hep-ph] .

[10] J. Engel, M. J. Ramsey-Musolf, and U. van Kolck, Prog. Part. Nucl. Phys. **71**, 21 (2013), arXiv:1303.2371 [nucl-th] .

[11] W. Dekens, J. de Vries, J. Bsaisou, W. Bernreuther, C. Hanhart, U.-G. Meißner, A. Nogga, and A. Wirzba, JHEP **07**, 069 (2014), arXiv:1404.6082 [hep-ph] .

[12] A. Wirzba, J. Bsaisou, and A. Nogga, Int. J. Mod. Phys. **E26**, 1740031 (2017), arXiv:1610.00794 [nucl-th] .

[13] J. Bsaisou, J. de Vries, C. Hanhart, S. Liebig, U.-G. Meissner, D. Minossi, A. Nogga, and A. Wirzba, JHEP **03**, 104 (2015), [Erratum: JHEP05,083(2015)], arXiv:1411.5804 [hep-ph] .

[14] T. Bhattacharya, V. Cirigliano, R. Gupta, H.-W. Lin, and B. Yoon, Phys. Rev. Lett. **115**, 212002 (2015), arXiv:1506.04196 [hep-lat] .

[15] S. Aoki *et al.* (Flavour Lattice Averaging Group), Eur. Phys. J. **C80**, 113 (2020), arXiv:1902.08191 [hep-lat] .

[16] C. C. Chang *et al.*, Nature **558**, 91 (2018), arXiv:1805.12130 [hep-lat] .

[17] M. Abramczyk, S. Aoki, T. Blum, T. Izubuchi, H. Ohki, and S. Syritsyn, Phys. Rev. **D96**, 014501 (2017), arXiv:1701.07792 [hep-lat] .

[18] M. Lüscher, JHEP **08**, 071 (2010), [Erratum: JHEP03,092(2014)], arXiv:1006.4518 [hep-lat] .

[19] A. Shindler, T. Luu, and J. de Vries, Phys. Rev. **D92**, 094518 (2015), arXiv:1507.02343 [hep-lat] .

[20] J. Dragos, T. Luu, A. Shindler, J. de Vries, and A. Yousif, (2019), arXiv:1902.03254 [hep-lat] .

[21] T. Izubuchi, H. Ohki, and S. Syritsyn, in *37th International Symposium on Lattice Field Theory (Lattice 2019) Wuhan, Hubei, China, June 16-22, 2019* (2020) arXiv:2004.10449 [hep-lat] .

[22] S. Syritsyn, T. Izubuchi, and H. Ohki (2019) arXiv:1901.05455 [hep-lat] .

[23] B. Yoon, T. Bhattacharya, V. Cirigliano, and R. Gupta, PoS **LATTICE2019**, 243 (2019), arXiv:2003.05390 [hep-lat] .

[24] T. Bhattacharya, V. Cirigliano, R. Gupta, E. Mereghetti, and B. Yoon, Phys. Rev. **D92**, 114026 (2015), arXiv:1502.07325 [hep-ph] .

[25] V. Cirigliano, E. Mereghetti, and P. Stoffer, Submitted to: JHEP (2020), arXiv:2004.03576 [hep-ph] .

[26] J. Dragos, T. Luu, A. Shindler, and J. de Vries, in *35th International Symposium on Lattice Field Theory (Lattice 2017) Granada, Spain, June 18-24, 2017* (2017) arXiv:1711.04730 [hep-lat] .

[27] M. D. Rizik, C. J. Monahan, and A. Shindler, (2020), arXiv:2005.04199 [hep-lat] .

[28] A. S. Gambhir, A. Stathopoulos, K. Orginos, B. Yoon, R. Gupta, and S. Syritsyn, *Proceedings, 34th International Symposium on Lattice Field Theory (Lattice 2016): Southampton, UK, July 24-30, 2016*, PoS **LATTICE2016**, 265 (2016), arXiv:1611.01193 [hep-lat] .

[29] S. Syritsyn, A. S. Gambhir, B. Musch, and K. Orginos, *Proceedings, 34th International Symposium on Lattice Field Theory (Lattice 2016): Southampton, UK, July 24-30, 2016*, PoS **LATTICE2016**, 176 (2017).