Hadronic contributions to the anomalous magnetic moment of the muon

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Motivation

The muon anomalous magnetic moment, $g-2$, has for many years exhibited a $3-4\sigma$ tension between the standard model (SM) theory calculation and the currently best experimental result, measured at BNL [1]. This provides an intriguing hint for possible physics beyond the SM. The current experimental precision of 0.54 ppm will be reduced by a factor of four in the coming years through the Fermilab E989 experiment [2]. In addition, an experiment with different systematics is planned in Japan at J-PARC [3]. The SM theory uncertainty currently stands at 0.37 ppm [4]. Therefore, in order to clarify the tension a similar reduction in theory uncertainty is needed.

The SM uncertainty is dominated by two non-perturbative QCD contributions, the hadronic vacuum polarization (HVP) and the hadronic light-by-light scattering (HLbL) with similar individual uncertainties. We discuss the current status and our plans to compute both at the desired precision from first principles in the next two sections. This letter describes our plans over the next five years and beyond. It builds on a multi-year effort [5–15] of high-precision first-principles calculations of both hadronic contributions to the muon $g-2$ by the RBC and UKQCD collaborations. Depending on experimental progress, the program discussed here may also help studies of the electron $g-2$.

Hadronic vacuum polarization

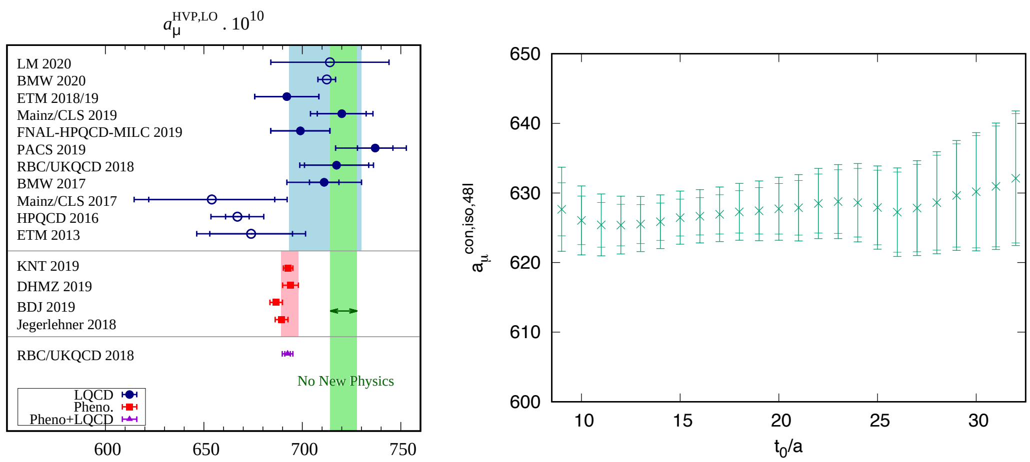

Figure 1: (Left) Current status of determinations of the leading-order HVP contribution. Adapted from Ref. [4]. (Right) Using the improved bounding method developed by our collaboration [9, 16], we can reduce the dominant uncertainty, the statistical noise for the light-quark isospin symmetric contribution, from $\approx 15\times10^{-10}$ to $\approx 2.5\times10^{-10}$ in the RBC/UKQCD 2018 48I result. Independence of the minimal Euclidean time t_0 for which bounds are used is expected and observed.

In order to reduce the HVP theory uncertainty to below the expected Fermilab experimental precision, we need to calculate it at a precision close to 0.2%. Figure 1 shows on the left side an overview of recent HVP theory results. The blue circles label lattice QCD calculations [5, 17–27] including our recent result RBC/UKQCD 2018 [5]. Results from the dispersive approach using e^+e^- annihilation experiments are shown as red squares [28–31]. In our latest work, we also introduced a new method to combine dispersive and lattice methods to suppress the respective dominant uncertainties. This result is shown as the purple triangle [5]. Finally, the ”no new physics” region shows the needed HVP value to bring theory and experiment of the $g-2$ in agreement. We note that most lattice results currently have a precision of $\approx 2\%$ with first results at sub-per-cent precision [26]. Since the first such calculation indicates a surprising tension with the dispersive results, it will be crucial for other high-precision results to appear in a timely manner and for those results to aim for statistics dominated error budgets. In the following, we outline our path towards 0.2% precision.

Our previous calculation [5] was performed at physical pion mass with both quark-connected and disconnected diagrams and included the dominant QED and strong isospin-breaking corrections. It also developed a highly efficient statistical sampling technique using the exact low-modes of the Dirac operator enabled by our locally-coherent eigenvector compression technique [32]. We also introduced the bias-free bounding method to reduce long-distance noise in the light-quark correlator [5, 20, 33]. We have since then improved the bounding method [9, 16] to reduce the dominant statistical uncertainty to a level similar to the uncertainties of the R-ratio calculations by using multiple-channel studies including two-pion and four-pion states. First results are shown on the right side of Fig. 1. This method produces as a byproduct also detailed information about the two-pion elastic scattering which in turn can be used to reduce uncertainties of the infinite-volume extrapolation. In order to reduce the uncertainties further we will first complete a calculation on a third lattice spacing at $a^{-1} = 2.7$ GeV. After completion of this effort, we will generate a series of at least four additional lattices at a pion mass of $m_\pi^H \approx 280$ MeV with kaon and Ω^- baryon masses kept equal to the physical pion-mass case. The difference between physical pion mass and m_π^H can then be calculated at three lattice spacings. Discretization errors are expected to cancel in this difference. At m_π^H, we then have excellent control of the continuum extrapolation including lattice spacings up to $a^{-1} \approx 5$ GeV. Progress outlined in a companion letter [34] may even allow for this to be performed at physical pion mass. The improved control of this extrapolation and studies using a variety of discretizations by the community will be crucial. For the infinite-volume extrapolation, we will measure explicitly on lattice volumes up to $(\approx 10 \text{ fm})^3$ aided by the ρ resonance studies and new methods such as Ref. [35]. We will add the currently neglected sub-leading QED and strong-isospin-breaking diagrams utilizing a beneficial cross-use of data generated for the HLbL. Finally, the precise calculation of isospin-breaking corrections will allow us to re-examine the use of τ decay data [10] to scrutinize data-driven approaches based on e^+e^- experiments and further reduce the HVP uncertainties.

Hadronic light-by-light scattering

The HLbL contribution to muon $g-2$'s current theoretical value is $9.2(1.8) \times 10^{-10}$ [4]. The value is obtained from a weighted average of phenomenology and lattice QCD determination. The phenomenology determination receives much higher weight, since the pure lattice result, which is obtained by our group with physical pion mass, is $7.87(3.06)_{\text{stat}}(1.77)_{\text{sys}}$ [7], dominated by the statistical error and less accurate than the phenomenology determination.

In this letter of interest, we will aim at reducing the total uncertainty of the lattice calculation to about 10%. To obtain the previous result, we performed the lattice QCD + QED calculation for the HLbL diagrams using the QED_L scheme, and lattices with several different volumes are used to extrapolated to the infinite volume. Our plan for the future is to focus on the QED_∞ method. By calculating the QED part of the HLbL diagram in the infinite volume semi-analytically, power-law suppressed finite volume errors are completely removed. Furthermore, we have discovered that some subtraction can be made to the QED kernel to reduce the statistical error and discretization error in the calculation [6]. In the recent work by the Mainz group [36], the QED_∞ approach is successfully applied in the HLbL calculation using a non-physical pion mass $M_\pi = M_K = 420$ MeV. To achieve the 10% accuracy goal, we will make progress in the following directions: compute the hadronic four point function more accurately with more advanced computational resources and better algorithms, e.g. low modes averaging and field sparsening; calculate the long distance of part of the hadronic four point function from the neutral pion-pole using the pion transition form factor, which can be calculated separately using lattice QCD; calculate the hadronic finite volume effects caused by the charged pion loop; explore better subtraction scheme to further improve the efficiency of the calculation; directly calculate all the sub-leading disconnected diagrams.

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