Searching for $\mu^- \to e^+$ Conversion at Upcoming Experiments and the Process of Radiative Muon Capture
Letter of Interest for Snowmass 2021

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This letter highlights the importance of a search for lepton number violation at the next generation of experiments with stopped muons, and, in particular, for the process of $\mu^- \to e^+$ conversion on nuclei. We also point out that improving the theoretical and experimental understanding of the process of radiative muon capture is critical for $\mu^- \to e^+$ conversion searches at upcoming experiments.

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MOTIVATION

The discovery of charged lepton flavor violation (CLFV) would be a smoking gun of physics beyond the Standard Model (SM) and is one of the most sought-after signals at the intensity frontier [1–3]. Historically, attention has focused on the lightest two lepton generations, making use of the fact that $m_\mu \gg m_e$ and the relatively long lifetime of the muon as compared to the tau. Much of the progress in the field has been driven by searches for $\mu \to 3e$ and $\mu \to e\gamma$ decays and $\mu \to e$ conversion on nuclei [1–6].

Searches for $\mu \to e$ conversion on nuclei include two possible detection channels: $\mu^- \to e^-$ and $\mu^- \to e^+$. The current best limits on $\mu \to e$ come from SINDRUM-II [7, 8]. Excitingly, the $\mu^- \to e^+$ channel not only violates flavor, but also total lepton *number* by two units, and is therefore a test of lepton number violation (LNV) that is complementary to neutrinoless double beta ($0\nu\beta\beta$) decay.

Although many theories of physics beyond the SM (BSM) point to $0\nu\beta\beta$ decay as the most sensitive probe of LNV, we remind the reader that in conventional Majorana models, this sensitivity is due exclusively to one entry of the neutrino mass mixing (i.e. PMNS) matrix $m_{\beta\beta} = m_{ee}$. Modern measurements of the neutrino mass hierarchy and mixing angles still allows for possibility of a vanishing $m_{\beta\beta}$ over a relatively wide range of parameter space [9], and it is therefore important to keep in mind that $0\nu\beta\beta$ is not *guaranteed* to discover LNV even if it occurs. The search for $\mu^- \to e^+$ therefore provides a complementary "safety net" since the LNV that necessarily mediates this process is in the off-diagonal $e\mu$ sector; to suppress both ee and $e\mu$ LNV without forbidding it altogether is a much more formidable task [10, 11]. This idea can be made technically natural by considering a gauged (and anomaly free) $L_e - L_\mu$ symmetry [12]. This flavour structure permits non-diagonal $e^\pm \leftrightarrow \mu^\mp$ transitions while forbidding $0\nu\beta\beta$ decay, since it violates $L_e - L_\mu$ by two units.

EXPERIMENTAL CONSIDERATIONS

Existing predictions for $\mu^- \to e^+$ conversion rates are significantly lower than rates predicted for $\mu^- \to e^-$ conversion. For that reason, detectors are typically optimized for the search in the $\mu^- \to e^-$ channel; therefore a simultaneous search for the $\mu^- \to e^+$ conversion requires a charge-symmetric detector.

To suppress background from radiative pion capture (RPC), experiments of the next generation, COMET and Mu2e, will be using a pulsed muon beam and the measurement window delayed with respect to the beam pulse. The μ^- lifetime in the stopping target material, as well as the time separation between the beam pulses, have therefore to be much longer than the charged pion lifetime. This requirement favors relatively light stopping target materials.

The experimental signature of a $\mu^- \to e^+$ conversion process is given by monochromatic ∼100 MeV/c positrons coming out of the nuclear target. The exact momentum depends on the choice of target material the muons are stopped in. However, the process of $\mu^- \to e^+$ conversion is not a coherent process, and a $\mu^- + (A, Z) \to e^+ + (A, Z - 2)_{GS}$ transition, where a daughter nucleus is produced in the ground state, has a much better chance to be separated from the background than transitions to the excited states of the $(A, Z - 2)$ nucleus. Thus optimization of the experimental sensitivity has to consider the probability of transition between the ground states of the initial and final state nuclei. This probability has been calculated in [13] for ^{27}Al, but not for other elements.

A potentially very important background from the process of radiative muon capture (RMC), $\mu^- + (Z, A) \to (Z-1, A) + \nu + \gamma(\to e^+e^-)$, could be reduced by a choice of the muon stopping target materials (Z, A) for which the corresponding daughter $(Z - 1, A)$ nucleus has a low nuclear binding energy with respect to $(Z - 2, A)$.

THEORETICAL CHALLENGES

The relative size of SM background contributions is an open question that must be answered to draw conclusions about LNV from Mu2e or experiments like it. Since the signal for $\mu^- \to e^+$ is positrons rather than electrons, the most formidable background in the $\mu^- \to e^-$ channel (muons decaying in orbit) disappears. What remains, however, are high energy photons that can produce electron-positron pairs either via in-medium pair production, or via virtually mediated "internal" pair production. After removing most of the RPC events via timing cuts, the majority of the background photons will be produced via

RMC. The critical important question to answer is what the shape and overall normalization of the positron spectrum is *near the end point*. It is this region that can overlap substantially with the e^+ signal region in Mu2e.

As of today, there is a significant gap in the literature surrounding RMC as it pertains to searches for $\mu^- \to e^+$. Most studies of nuclear RMC rely on approximations (such as the closure approximation) that are unreliable near the end point [14–16]. Moreover, few studies focus on the end-point at all, focusing rather on the total integrated rate; as emphasized above this is the opposite hierarchy of need for a $\mu^- \to e^+$ experiment. There is, consequently, a pressing need for better predictions of both the real photon spectrum from RMC on medium heavy nuclei (and specifically ^{27}Al) [17], as well as the spectrum of the resultant positrons [18].

Another pressing issue is the evaluation of nuclear matrix elements for the ground state → ground state nuclear transition that accompanies $\mu^- \to e^+$[10]. Sharing close analogies with $0\nu\beta\beta$ and $2\nu\beta\beta$ decay the process $\mu^- \to e^+$ involves the insertion of two electroweak charged currents in the nucleus. Rather than producing two charged leptons as in $0\nu\beta\beta$, in $\mu^- \to e^+$ one negatively charged lepton is absorbed, while one positively charged lepton is emitted. Nevertheless, the kinematics of $\mu^- \to e^+$ differ substantially from those of either double-beta decay process, and a dedicated industry focused on nuclear matrix elements will be required to convert future (non-)observations of $\mu^- \to e^+$ into (exclusion limits on) LNV couplings within a specific UV-complete model of new physics[13].

WHAT CAN BE LEARNED FROM THE PUBLISHED DATA?

The best experimental limit on $\mu^- N(Z,A) \to e^+ N(Z-2,A)$ has been published by the SINDRUM-II's experiment in 1998: $\mathrm{Br}(\mu^-\mathrm{Ti} \to e^+\mathrm{Ca}) < 1.7 \cdot 10^{-12}(\mathrm{GS})$ [7]. After that, SINDRUM-II collected data on an Au target with significantly higher number of stopped muons, but only published a limit on the $\mu^- N(Z,A) \to e^- N(Z,A)$ conversion search: $\mathrm{Br}(\mu^-\mathrm{Au} \to e^-\mathrm{Au}) < 7 \cdot 10^{-13}$ [8]. In the same paper, along with the electron momentum spectrum used to set the limit, SINDRUM-II published the positron spectrum, a detailed examination of which the leads to rather interesting observations [19].

The positron momentum spectrum is dominated by RMC. The assumption that the RMC contribution can be described by the closure approximation results in a clear excess of events right above the kinematic endpoint of the RMC spectrum. The excess has a form of a bump with the width consistent with the detector resolution. A naive estimate of the probability of a statistical background fluctuation gives $P < 1 \times 10^{-9}$.

An exotic explanation of a bump as a signal from $\mu^-\mathrm{Au} \to e^+\mathrm{Ir}$ conversion seems unlikely: the position of the bump is about 1 MeV/c, or about 4σ, lower than the expected signal position. In principle, the excess could be explained by an exclusive RMC-induced transition $\mu^- + \mathrm{Au} \to \nu + \gamma(\mathrm{e}^+\mathrm{e}^-) + \mathrm{Pt}^*$ into an exclusive excited state of the daughter nucleus (Pt*). The needed strength of such a transition is below the sensitivity level of published RMC measurements, and therefore this explanation cannot be excluded. It is worth noting, that to get a satisfactory background description, a similar assumption has been made in [7]. Currently, not much is known about such transitions, and their exclusion could lead to a significant tension between the SINDRUM-II positron data on Au target and non-exotic explanations of those data. On the other hand, their presence could significantly modify the SM background in $\mu^- \to e^-$ channel. This highlights the importance of improving the theoretical understanding of RMC for the $\mu^- \to e$ conversion searches at the upcoming experiments.

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