Physics of muonium and muonium oscillations

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Precision studies of a muonium, the bound state of a muon and an electron, provide access to physics beyond the Standard Model. We propose that extensive theoretical and experimental studies of atomic physics of a muonium, its decays and muonium-antimuonium oscillations could provide an impact on indirect searches for new physics.

An especially clean system to study BSM effects in lepton sector is muonium M_μ, a QED bound state of a positively-charged muon and a negatively-charged electron, $|M_\mu\rangle \equiv |\mu^+ e^-\rangle$. The main decay channel of the state is driven by the weak decay of the muon. The average lifetime of a muonium state τ_{M_μ} is thus expected to be the same as that of the muon, $\tau_\mu = (2.1969811 \pm 0.0000022) \times 10^{-6}$ s [1], apart from the tiny effect due to time dilation, $(\tau_{M_\mu} - \tau_\mu)/\tau_\mu = \alpha^2 m_e^2/(2m_\mu^2) = 6 \times 10^{-10}$. Such a lifetime, in principle, is rather long to allow for precision measurements of muonium's atomic and particle physics properties [2]. In this Letter of Interest (LOI) we will argue that a robust theoretical and experimental research program in muonium physics will help us better understand both precision Standard Model (SM) physics and place competitive constraints on New Physics (NP) interactions.

I. MUONIUM: ATOMIC PHYSICS

Muonium's structure is similar to that of a hydrogen without the added complexity of a proton, both muon and electron are point-like, which helps in interpretation of atomic structure measurements. Just like a positronium or a Hydrogen atom, muonium could be produced in two spin configurations, a spin-one triplet state called *ortho-muonium*, and a spin-zero singlet state called *para-muonium*.

Atomic calculations of muonium properties are important and could be related to other systems. Measurements of its hyperfine splitting $\Delta\nu_{\rm HFS}$, its Zeeman effect, the $1s-2s$ frequency difference $\Delta\nu_{1s2s}$ could be useful in connection with similar computations in Hydrogen without the complications of strong interactions and for searches for New Physics. For example, the light-by-light (LBL) contribution to the hyperfine splitting in muonium is similar to the LBL contribution to $(g-2)$. A three-loop correction has recently been computed [3] with hadronic LBL scattering contribution computed earlier [4].

II. MUONIUM: DECAYS

Flavor-changing neutral current (FCNC) interactions serve as a powerful probe of physics beyond the standard model (BSM). Since no local operators generate FCNCs in the Standard Model at tree level, New Physics degrees of freedom can effectively compete with the SM particles running in the loop graphs, making their discovery possible. This is, of course, only true provided beyond the Standard Model (BSM) constructions include flavor-violating interactions. In order to probe those we consider muonium decays and oscillations.

Denoting the "muon quantum number" by L_μ, FCNC decays of a muonium would probe $\Delta L_\mu = 1$ interactions. The effective Lagrangian, $\mathcal{L}_{\rm eff}^{\Delta L_\mu=1}$, can then be divided into the dipole part, $\mathcal{L}_D$, and a part that involves four-fermion interactions.

$$\mathcal{L}_{\rm eff} = \mathcal{L}_D + \mathcal{L}_{\ell q} + ... \tag{1}$$

Here the ellipses denote effective operators that are not relevant for the following discussion. The dipole part in Eq. (1) is usually written as

$$\begin{aligned}\mathcal{L}_D = & - \frac{m_{\ell_1}}{\Lambda^2}\Big[\Big(C_{DR}^{\ell_1\ell_2}\ \overline{\ell}_2\sigma^{\mu\nu}P_L\ell_1 \\ & + \ C_{DL}^{\ell_1\ell_2}\ \overline{\ell}_2\sigma^{\mu\nu}P_R\ell_1\Big)F_{\mu\nu} + h.c.\Big],\end{aligned} \tag{2}$$

where $P_{\rm R,L} = (1 \pm \gamma_5)/2$ is the right (left) chiral projection operator. The Wilson coefficients would, in general, be different for different leptons ℓ_i.

The four-fermion dimension-six lepton-quark Lagrangian takes the form [5]

$$\begin{aligned}\mathcal{L}_{\rm eff}^{\Delta L_\mu=1} = & -\frac{1}{\Lambda^2}\sum_f \Big[\ \Big(C_{VR}^f\ \overline{\mu}_R\gamma^\alpha e_R + C_{VL}^f\ \overline{\mu}_L\gamma^\alpha e_L\Big)\, j_\alpha^V \\ & + \ \Big(C_{AR}^f\ \overline{\mu}_R\gamma^\alpha e_R + C_{AL}^q\ \overline{\mu}_L\gamma^\alpha e_L\Big)\, j_\alpha^A \\ & + m_e m_f G_F\ \Big(C_{SR}^f\ \overline{\mu}_R e_L + C_{SL}^f\ \overline{\mu}_L e_R\Big)\, j^S \\ & + m_e m_f G_F\ \Big(C_{PR}^f\ \overline{\mu}_R e_L + C_{PL}^f\ \overline{\mu}_L e_R\Big)\, j^P \\ & + m_e m_f G_F\ \Big(C_{TR}^f\ \overline{\mu}_R\sigma^{\alpha\beta} e_L + C_{TL}^f\ \overline{\mu}_L\sigma^{\alpha\beta} e_R\Big)\, j_{\alpha\beta}^T\Big],\end{aligned} \tag{3}$$

where $j_\alpha^V = \overline{f}\gamma_\alpha f$, $j_\alpha^A = \overline{f}\gamma_\alpha\gamma_5 f$, $j^S = \overline{f}f$, $j^P = \overline{f}\gamma_5 f$, and $j_{\alpha\beta}^T = \overline{f}\sigma_{\alpha\beta}f$ are the fermion currents with f representing fermions that are not integrated out at the the muonium scale. The subscripts on the Wilson coefficients are for the type of Lorentz structure: vector, axial-vector, scalar, pseudo-scalar, and tensor. The Wilson coefficients would in general be different for different fermions f.

The effective Lagrangian of Eq. (1) could be probed in muon decays such as $\mu \to e\gamma$ or $\mu \to 3e$, etc. These decays receive contributions from many possible operators in Eqs. (2) and (3), making it difficult to identify the Lorentz structure of possible NP interactions. This problem could be alleviated by studying decays with constrained kinematics [5, 6]. Studying decays of para- and ortho-muonia $M_\mu^V \to e^+e^-$, $M_\mu^V \to 3\gamma$, and $M_\mu^P \to \gamma\gamma$ will only select particular combinations of the operators in Eq. (3), making possible identification of NP interactions.

III. MUONIUM-ANTIMUONIUM OSCILLATIONS

In the presence of flavor-violating interactions muonium $|M_\mu\rangle \equiv |\mu^+e^-\rangle$ could oscillate into an antimuonium $|\overline{M}_\mu\rangle \equiv |\mu^-e^+\rangle$ state. While such processes are strongly suppressed by neutrino masses in the Standard Model, plenty of BSM scenarios boast much larger transition rates [7]. A mere presence of $\Delta L_\mu = 2$ interactions leads to the fact that muonium flavor eigenstates are no longer its mass eigenstates, leading to the time-dependent oscillations of flavor states, and generation of the mass and lifetime splittings of its mass eigenstates, which we denote Δm and $\Delta\Gamma$, respectively.

Denoting an amplitude for the muonium decay into a final state f as $A_f = \langle f|\mathcal{H}|M_\mu\rangle$ and an amplitude for its decay into a CP-conjugated final state $\overline{f}$ as $A_{\bar{f}} = \langle \overline{f}|\mathcal{H}|M_\mu\rangle$, we can write the time-dependent decay rate of M_μ into the $\overline{f}$ [8],

$$\Gamma(M_\mu \to \overline{f})(t) = \frac{1}{2} N_f \left|A_f\right|^2 e^{-\Gamma t} \left(\Gamma t\right)^2 R_M(x,y), \quad (4)$$

where N_f is a phase-space factor and $R_M(x,y)$ is the oscillation rate,

$$R_M(x,y) = \frac{1}{2}\left(x^2+y^2\right), \quad (5)$$

where $x = \Delta M/\Gamma_{M_\mu}$ and $y = \Delta\Gamma/2\Gamma_{M_\mu}$. Since ΔM and $\Delta\Gamma$ are dominated by New Physics interactions [8], while Γ_{M_μ} is given by the SM interactions, $x, y \ll 1$. Thus, oscillating functions can be expanded in x and y resulting in a power-law dependence displayed in Eq. (5).

The most recent experiment studying $M_\mu - \overline{M}_\mu$ oscillations was done in the 1990s [9], and was statistics limited by the available μ^+ flux. It could not study the time-dependence of Eq. (4). A new experiment with sensitivity improved by 2-3 orders of magnitude could exploit the time evolution of the $M_\mu - \overline{M}_\mu$ conversion process.

Integrating over time and normalizing to $\Gamma(M_\mu \to f)$ we get the probability of M_μ decaying as $\overline{M}_\mu$ at some time $t > 0$ [8],

$$P(M_\mu \to \overline{M}_\mu) = \frac{\Gamma(M_\mu \to \overline{f})}{\Gamma(M_\mu \to f)} = R_M(x,y). \quad (6)$$

Using the data from [9] to place constraints on the oscillation parameters, one must take into to account the fact that the set-up described in [9] had muonia propagating in a magnetic field B_0. This magnetic field suppresses oscillations by removing degeneracy between M_μ and $\overline{M}_\mu$. It also has a different effect on different spin configurations of the muonium state and the Lorentz structure of the operators that generate mixing [10–12]. Experimentally these effects could accounted for by introducing a factor $S_B(B_0)$. The oscillation probability is then [9],

$$P(M_\mu \to \overline{M}_\mu) \leq 8.3 \times 10^{-11}/S_B(B_0). \quad (7)$$

A more thorough analysis of the effects of magnetic fields would be desirable for better constraints on the oscillating parameters.

One can use Ref. [8] to relate x and y to the BSM scale Λ (or the Wilson coefficients C_i of BSM operators) and constrain their values from the experimental data. Since both spin-0 and spin-1 muonium states were produced in the experiment [9], one should average the oscillation probability over the number of polarization degrees of freedom,

$$P(M_\mu \to \overline{M}_\mu)_{\rm exp} = \sum_{i=P,V} \frac{1}{2S_i+1} P({M_\mu}^i \to {\overline{M}_\mu}^i), \quad (8)$$

where $P(M_\mu \to \overline{M}_\mu)_{\rm exp}$ is the experimental oscillation probability from Eq. (7).The values of $S_B(B_0)$ are given in Table II of [9]. As can be seen [8], even 20 year old experimental data provide constraints on the New Physics scale that are comparable to those probed by the LHC program of the order $\Lambda \sim 5$ TeV. An improvement of sensitivity by two orders of magnitude will allow constraint specific models of New Physics better then the combinations of other measurements. We urge our experimental colleagues to further study muonium-antimuonium oscillations. For this, a pulsed μ^+ source could significantly reduce the ratio of the potential $\bar{M}$ signal to μ^+ and M decay related background [2].

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