

# Need for amplitude analysis in the discovery of new hadrons

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This Letter of Interest highlights the need for development of comprehensive amplitude analysis methods to further our understanding of hadron spectroscopy. Reaction amplitudes constrained by first principles of  $S$ -matrix theory and by QCD phenomenology are needed to extract robust interpretations of the data from experiments and from lattice calculations.

In the last two decades, high-energy physics experiments have delivered a lot of unexpected exotic hadron resonances, that challenge the minimal quark model lore of baryons with three quarks and mesons with a quark-antiquark pair. Candidates for tetraquarks, pentaquarks, molecules, and for hadrons with gluonic degrees of freedom have been found [1, 2]. Establishing the existence of isolated resonances that go beyond the minimal quark model is just the first step: one needs to identify the complete multiplets and study the differences and similarities among their members. This would provide insights into the nature of exotic resonances and the inner workings of QCD in the nonperturbative regime. However, a comprehensive and consistent picture of this sector of the spectrum is still missing. Many of these resonance candidates have been seen in just a single production and decay channel. The analyses at lepton colliders have been limited by statistics so far. Moreover, measurements often face complications due to the presence of multibody final states, which makes a model-independent determination of an exotic candidate difficult.

Despite tremendous progresses in understanding gauge theories, an analytic solution of QCD in the nonperturbative regime will not be available in the foreseeable future. At the moment, Lattice QCD represents the most rigorous tool to calculate observables from first principles, albeit numerically [3]. However, it does not answer how the specific properties of QCD, as confinement and mass generation, emerge. Ultimately, it does not explain *why* quarks and gluons organize themselves in the hadron spectrum in the way we observe it. For this, using other approximate tools (such as functional methods [4]) and models of QCD (as the quark model, or the holography-inspired description [5]) is required. Together with these top-down approaches, bottom-up strategies are also feasible. We know indeed that any reaction amplitude in QCD must satisfy a set of general principles, such as unitarity, analyticity, crossing and Lorentz symmetries, as well as the specific symmetries of the strong interactions [6]. One can thus write ansätze that follow these principles as much as possible, at least in a given kinematical domain, and fit to data. If the amplitude model space is large enough, the resonance properties obtained will be as unbiased as possible.

Once the theoretical modeling of the reaction amplitude has been achieved, it can be combined with a sound statistical data analysis. Monte Carlo approaches allow for a systematic analysis of the statistical uncertainties [7]. The model dependencies can be reduced with statistical learning (cross-validation, ridge methods, stability selection and artificial neural networks) [8]. Moreover, one can use clustering methods to separate the physical resonances

from the artifacts of the amplitude parametrizations [9]. All these studies allow one to give a robust determination of resonances and of their properties. These analyses are computationally expensive and require high-performance computing resources, but they will become mandatory for the interpretation of future high-precision data.

The Joint Physics Analysis Center (JPAC) is an example of a collaboration between theorists and experimentalists aiming at developing amplitude analysis tools for hadron spectroscopy. The methods discussed above require complementary sets of skills in QCD, reaction theory, computer science, and experimental data analysis. The group has a strong record of interactions with experiments: JPAC members have contributed to analyses by BaBar [10], BESIII [11], CLAS [12, 13], COMPASS [14, 15], GlueX [16], and LHCb [17], and to several proposals of future spectroscopy experiments.

Three main research lines can be identified:

(i) *amplitude analysis formalism*. Multibody decays of particles with spin contains theoretical subtleties that can be overlooked and affect the result of experimental analyses. These have been addressed in several recent publications [18]. Phenomenological studies of three-body decays of light mesons [19], and specifically the  $\eta \rightarrow 3\pi$  which gives access to the  $u$ - $d$  isospin breaking [20], are another area of intense studies. The method adopted has been validated against  $\pi\pi$  scattering data [21]. This techniques can also be extended to charm meson decays, which are of interest for flavor physics. Further advancements in extracting the hadron spectrum from Lattice QCD requires state-of-the-art three-to-three scattering amplitudes, studied in [22].

(ii) *modeling of production mechanisms*. Analyticity provides strong constraints for the asymptotic behavior of amplitudes. This allows to give predictions for photoproduction of mesons at high energies [23]. In particular, the predictions in [24] are currently being used to shape the spectroscopy program at the forthcoming Electron Ion Collider [25], or other future photon-hadron facilities. Sensitivity to light exotics and heavy pentaquarks in present photoproduction experiments has also been explored [26]. Pion and kaon beams are studied in [27]. The asymptotic regime is analytically connected to the resonance region, and this information can be used to further constrain the available low-energy partial wave analyses [28].

(iii) *resonance studies*. The extraction of resonance properties, especially when it comes to exotic states, is of high importance because it sheds light on the nonperturbative aspects of QCD. For example, the recent study of the  $a_1(1260)$  in  $\tau$  lepton decay resolved the discrepancies in its properties extracted from different reactions [29]. The searches for mesons with explicit gluonic degrees of freedom has motivated several experiments. Two light candidates close in mass were identified, while only one was expected from theory. This longstanding puzzle was resolved in [9], where it was shown that both signals can be explained by a single state if a proper amplitude study is performed. A detailed study of the lineshape of resonances using minimally biased amplitudes gives insights into their nature: while limited statistics prevented to draw strong conclusions for the four-quark  $Z_c(3900)$  candidate [30], the data allowed to conclude that the intriguing LHCb pentaquark candidate  $P_c(4312)$  is a virtual state [31]. The strange baryon spectrum, together with their Regge trajectories and also with the ones of  $N^*$  and  $\Delta^*$ , was studied in [32].

To push forward our understanding of the hadron spectrum, precise measurements are needed, as well as searches of states in production mechanisms complementary to the ones already explored. LHCb and BESIII are recording data with unprecedented statistics, and Belle-II, GlueX and CLAS12 have just started operating and promise exciting results in the near future. The plans for next-generation facilities include Electron Ion Colliders, a  $p\bar{p}$  machine at the charm energies ( $\bar{P}$ ANDA), and COMPASS++/AMBER able to run with a variety of beam species. In addition, lattice QCD is expected to deliver new observables, whose extraction benefits from proper amplitudes. In turn, these physics programs require better amplitude analysis tools. For these analyses to be successful, we stress the importance of tight collaborative efforts between theorists and experimentalists, in the same spirit as the ones pioneered by JPAC.

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