Letter of interest for Hadron Spectroscopy with Lattice QCD

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Abstract: Most of the open questions in hadron spectroscopy are related to hadronic resonances and exotic hadrons. Experiments discovered many hadrons with exotic minimal quark content $\bar{q}\bar{q}qq$ or $\bar{q}qqqq$, but all of those decay strongly via one or multiple decay-channels. This presents a challenge for their rigorous theoretical study based on first-principles Quantum ChromoDynamics on the lattice. Several important problems in this respect have already been solved. We propose to study some challenges by the lattice and analytical methods that would help to resolve the nature of these interesting hadronic states. Some initial thoughts are listed.

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Hadrons are composed of quarks and gluons, which are held together by the strong interaction. This interaction has large strength at small energies which invalidates perturbative theoretical study of hadronic properties. Lattice QCD is widely used non-perturbative and systematically improvable method, based directly on the fundamental theory.

The majority of hadrons decay quickly via the strong interaction, which also applies to all experimentally discovered exotic hadrons. Hadrons that decay strongly, present a non-trivial task for lattice QCD since they are not eigenstates of QCD. Below we briefly list the solved problems and future challenges in this respect[2].

Problems that have been solved by a number of lattice studies (reviewed in e.g. [1, 2])

1. Masses and transition matrix elements of hadrons that cannot decay strongly
2. Masses and widths of resonances that decay strongly only to one pair of spin-less hadrons

Future challenges and problems that need more consideration

The majority of hadrons are hadronic resonances that can strongly decay to multiple final states. This applies to almost all experimentally discovered exotic hadrons. Most of challenges are still open in this case (see also USQCD white paper [3]):

1. Consider channels and energy regions that feature exotic hadrons and can be reliably investigated with both lattice QCD and experiment. Those are regions below or near the lowest strong decay threshold for lattice QCD, but only few exotic hadrons are found there. The existence of the $bb\bar{u}\bar{d}$ state is now firmly established on the lattice (e.g. [4, 5]), but the prospects for establishing it in experiment is not good. Experiments found a large number of exotic hadrons in energy regions that are challenging for rigorous lattice study. It would be valuable to identify certain exotic hadrons that could be studied well on both sides.
2. Determine masses and widths of hadronic resonances that strongly decay to two or three final states with two spin-less hadrons. This requires the determination of scattering matrices for coupled-channel scattering, which was mainly accomplished by the Hadron Spectrum Col. for hadrons composed of u, d, s (e.g. [6, 7, 8]). Hadrons with heavy quarks generally couple to more channels and scattering matrix for only one system was determined [9].

   Explore hadronic resonances or near-threshold states that strongly couple to a pair of hadrons that carry non-zero spin, where dynamical mixing of partial waves is taken into account (e.g. [10]). Channels with non-zero spin present additional challenges since a number of nearly-degenerate eigenstates appear due to various spin projections.
3. Investigate resonances that lie well above the lowest threshold and have more than three strong decay modes ($Z_c(4430)$, P_c, Z_b, $X(6900)$, glueballs, ..): it is unlikely that these could be treated rigorously in the near or even more distant future. Simplifications will likely be needed and some of those are also listed below. It would be valuable to establish which channels could be treated as decoupled to a reasonable approximation.
4. Analytically or numerically determine the effect of omitted $Q\bar{Q}$ annihilation for quarkonia or quarkonium-like states.

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[2] The list mostly refers to generalizations of the Lüscher's method for extracting the scattering matrices.

5. Improve analytical methods to extract poles of scattering amplitudes from eigen-energies [11].
6. Vary the quark masses in order to explore the position of the resonances with respect to the position of thresholds and to investigate the structure of these resonances (e.g. [12]).
7. Try to find a reliable criterion on the importance of molecular and diquark configurations within a hadron to learn about its structure. This is complicated by the fact that two-meson and diquark-antidiquark structures in a meson are related via the Fierz transformation.
8. Study the simplest channels with three-hadron states and resonances that decay to three-hadrons. Contact three body interaction was determined only for the $\pi\pi\pi$ system [13, 14] from the lattice eigenenergies [15, 16] based on the recently developed formalism (e.g. [17, 18, 19]).
9. Try to rigorously incorporate three-hadron decays to specific channels, where these were omitted (for example $X(3872) \to J/\psi\pi\pi$). There are many interesting resonances, which have two-hadron and three-hadron decay modes. Sometimes, it is challenging to handle them rigorously. Explore alternative approaches (e.g. [20]) and investigate, whether certain simplifications can be done in the extraction of the physical observables from lattice data.
10. Determine the yet unknown Born-Oppenheimer potentials for systems with two heavy particles and light degrees of freedom. Consider the effect of strong decay in hybrid potentials. Many potentials for $QQ\bar{q}\bar{q}'$ systems have already been determined (e.g. [21]), while only few were determined for conventional [22, 23] and exotic [24] systems $\bar{Q}Q\bar{q}q'$.
11. Establish analytical methods for determining Born-Oppenheimer potentials at very small distances. Those are needed since potentials can be determined on the lattice only for separations larger than the lattice spacing.
12. Determine matrix elements for electro-weak transitions between a resonance and a stable state (e.g. [25]), where only EM transition $\pi \leftrightarrow \rho$ was determined in practice [26]. Using different methods, explore the feasibility of a rigorous extraction of the electro-weak transition matrix elements between two resonances (e.g. [27]), which probe resonance structure.
13. Explore, whether the information about the overlaps of the eigenstates to various operators can be useful to study the structure of the particles associated with these eigenstates. Currently, this information is used qualitatively rather than quantitatively to obtain various Fock-components. In this connection, we note that, in the HALQCD method, the Bethe-Salpeter wave functions and the hadron-hadron potential are extracted from overlaps [28].
14. Try to extract scattering amplitudes from finite-volume spectral functions based on the LSZ formalism as proposed in [29].
15. The major problem for a lattice study of a resonance that lies high above the threshold is that one needs to consider also the whole energy region that lies below it. This makes rigorous studies of interesting states like $Z_c^+(4430)$ or $X(6900)$ almost impossible. It would be very valuable to find some approach that could address just a certain higher-lying energy region. This does not seem viable with currently used lattice methods.

Resolving some of the listed challenges would improve our understanding of the experimentally observed conventional and exotic hadrons. It will help to determine which experimental peaks in the cross-section owe their existence to the nearby thresholds, diquark clustering or conventional hadrons. The predictions of yet-unobserved states will guide future experiments.

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