Entanglement, Spacetime, and Holography: LOI for Snowmass TF10

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August 2020

The connection between quantum information science (QIS) and quantum gravity has been a close and fruitful one, particularly in the past decade. Quantum error correction seems critical to a detailed understanding of the bulk/boundary correspondence in AdS/CFT. Entanglement seems to be the manner in which spacetime itself is stitched together. Entropic relations seem to define the rules of which quantum states are dual to semiclassical spacetime and which are not. We expect this field to continue to progress over the next several decades, with many more surprises to come.

Some of the ideas that we are particularly excited about are as follows:

Spacetime reconstruction from Entanglement

Much progress has been made in reconstructing aspects of the bulk spacetime theory from the boundary data in the context of AdS/CFT, with significant work focusing on reconstructing operators perturbatively on top of a fixed bulk spacetime geometry, see for example [1] or [2]. Recently, it has also been found that in 4+ dimensional spacetimes the bulk spacetime metric itself can be reconstructed with the knowledge of area variations of minimal bulk dimension-2 surfaces [3]; in 4 dimensions these surfaces are the Ryu-Takayanagi surfaces or Hubeny-Rangamani-Takayanagi surfaces that compute entanglement entropies. In other $d > 4$ the $2 - d$ surfaces are still sufficient to reconstruct the metric but do not compute entanglement themselves.

This method may be extendable to further cases, e.g. using either lower or higher dimensional minimal surfaces, and in lower dimensional spacetimes. There are also some genericity assumptions that can potentially be removed/eased. Other extensions include applicability to non-classical geometries, and numerical procedures to implement the algorithm as in [4]. In the end, if it can be established that knowledge of entanglement entropies of all subregions of the boundary is sufficient to reconstruct the bulk spacetime in full generality, this would go a long way towards rigorizing the statement that entanglement emerges and defines spacetime, at least in AdS/CFT.

As a related direction, the extension of holographic/quantum information theoretic techniques should be extended to non-relativistic spacetimes, such as warped CFT spacetimes, Lifshitz, and Schrodinger spacetimes, as well, to make contact with predictions for non-relativistic boundary theories.

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Entanglement Entropy constraints from and for Holography

The set of entanglement entropy inequalities that a quantum state in a quantum field theory obeys indicate whether that state can be dual to a semi-classical spacetime that obeys the Ryu-Takayanagi formula. Importantly, there are inequalities that are obeyed by entanglement entropies of these so-called "holographic states" that are not obeyed by all quantum states; see for example [5, 6, 7, 8]. An important development in this area has been the development of the holographic entropy cone in [7] and related objects such as the holographic entropy arrangement [9].

In the context of the holographic entanglement entropy, the key technological innovation has been the ability to convert entanglement entropy inequalities from linear algebraic relations to geometric/graph theoretic constraints, thus making these inequalities much easier to prove. Recently, there has been a push to generalize these new proof techniques to increasing generalizations of geometry and graph theory, with the first of these being the extension to hypergraphs to be able to better represent for example GHZ-type entanglement that is not representable by graphs/holographic states [10, 11, 12]. Progress in this direction leads naturally to investigations into whether further generalizations can lead to a characterization of the set of either all stabilizer states, or that of all quantum states, with potential generalizations involving the assignment of directions to edge graphs, the inclusion of topological data in the forms of linked but not edge-wise connected graphs, and combinations thereof.

Novel Entanglement Measures from and for Holography

Entanglement measures are an important aspect of quantum information theory as they give resource estimates that quantify the ability of certain quantum states/circuits to perform specific quantum tasks. It is natural to investigate what entanglement measures, beyond entanglement entropy itself, are well-suited to characterize the ability of holographic states/dynamics to perform certain tasks. Some progress in this direction has been the entanglement of purification program originated by [13, 14]. Further research in this direction has included multipartite generalizations [15, 13], efforts towards proving the core conjecture [16] and relations of the entanglement wedge cross section to other conjectured entanglement measures [17]. Further investigations into the entanglement of purification could be very interesting, as well as the development of genuinely multipartite entanglement measures motivated by the bulk spacetime geometry of holographic states.

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