Benchmarking Quantum Platforms with High Energy Physics

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Many problems of physical interest appear eventually amenable to a quantum advantage, but the resources required remain to be determined. In this LOI, we discuss benchmark calculations for a sequence of models (*Kogut's ladder*) that would comprehensively test and direct the development of quantum hardware and algorithms.

I. INTRODUCTION

Quantum computers promise solutions to problems limiting classical computers in many aspects of high energy physics, in particular finite density and real time evolution [1]. While recent results have demonstrated quantum advantage in a specially designed problem [2], calculations of interest to high energy physics appear to require more resources. In this LOI, we discuss how the history of success in co-designing lattice field theory problems with state-of-the-art classical hardware can be repeated with quantum hardware, emphasizing two points: model ladders and hardware evaluation.

II. KOGUT'S LADDER

The nearly fifty-year development of lattice field theory has both shaped and been shaped by the existing classical computers available. While Wilson's proposal of lattice regularization for QCD occurred in 1973 [3], it would be nearly three decades of feedback between theory, algorithms and hardware before accurate QCD calculations with dynamical fermions were being performed [4]. This was possible because instead of waiting for the naive resource estimates for QCD, at every instant other quantum field theories were experimented with and used to benchmark hardware. Taking the past as prologue, the same road map through model space could prove powerful for quantum computing. In review articles in 1979 and 1983, Kogut [5, 6] discusses a number of models of increasing complexity, in what became known as *Kogut's Ladder*. At the ladder's top is 3+1d $SU(N)$ gauge theories with matter – the standard model. Lower rungs require few resources by reducing dimensionality, degrees of freedom, and symmetries – providing intermediate milestones for quantum platforms. For each model, observables of increasing complexity should be studied, since they test different capabilities.

The lowest rung is the set of 1+1d spin chains such as the transverse Ising model. These models naturally align with the currently available architecture. Any 1+1 theory (e.g. Thirring and Schwinger models, the Abelian Higgs models, 1+1 QCD) can be written as a spin chain. Many models can be computed analytically, providing exact comparisons. These models have seen extensive testing on current platforms (references with particular high energy physics emphasis include [7–11]) and initial benchmarks of methods and hardware have been obtained [12, 13]. Further proposals requiring larger resources exist [14–22]. Finding efficient ways to extract classical-inaccessible observables should be a prime focus [23–26].

Advancing upward, one moves to 2+1d where complications arise in representing fermions efficiently[27–30], gauge fields become dynamic, and renormalization must be performed. These theories do not necessary map easily onto existing architectures, and communication between hardware and users will be crucial. The simplest model is perhaps the $\mathbb{Z}_2$ gauge theory which is dual to the 2+1d transverse Ising model. This duality reduces gauge redundancies reducing quantum resources required to simulate [31, 32]. While moving up the ladder, it will be important to compare the efficiency of truncated character expansions and approximations by discrete subgroups. As quantum resources proliferate, larger $\mathbb{Z}_n$ become accessible where the limit $n \to \infty$ corresponds to $U(1)$. Other digitizations of $U(1)$ exist[30, 33–38], which make different approximation to the continuous gauge theory. With sufficient connectivity, pure fermionic theories could also be simulated[39–43]. One also expects scalar field theory calculations would be performed[44]. Additionally, new observables like shear viscosity can be computed in 2+1d. Limited connectivity favors using open boundary conditions over periodic boundary conditions, but this complicates analysis of systematics.

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The final set of rungs introduces new complications. Non-abelian theories like $SU(N)$ will require longer circuit depth than $\mathbb{Z}_N$ and $U(1)$ for the same number of qubits [45–47]. Mapping 3+1d or higher dimensional lattice theories onto 2d architecture will require further developments. Integrating dynamical fermions together with the gauge fields will demand further resource and algorithmic improvements [48, 49].

III. HARDWARE EVALUATION

There is no single, quantitative metric in quantum computing. Even knowing a long list of metrics for a given hardware (e.g. number of qubits, coherence time, connectivity, error rate) cannot easily be mapped into a metric expressing how well a given HEP calculation will perform − there is no substitute for explicit implementation to test the domain-specific ability of the hardware. Without this concrete, constant feedback from physicists, efforts to improve quantum hardware may focus on issues that at a given moment do not represent our most pressing issues. In particular, common algorithms in HEP require deep circuits scaling with N qubits like $\mathcal{O}(N^3)$ or worse.

Kogut's ladder is structured in terms of model milestones with some forseeable problems, but doubtlessly unanticipated obstacles will arise along the climb. Furthermore, for a given model there are numerous questions that must be addressed e.g. digitization, state preparation [23, 40, 44, 50–55], extraction of observables. We expect that unimagined theoretical and algorithmic developments will be required to overcome these. If the history of lattice field theory is a guide, these results will radically change the ultimate resources requirements needed for a given calculation − centuries becoming decades becoming years.

While we lack answers to the following questions, they represent areas where exciting advances would leading to new insight in the true resource requirements:

- *Will the symmetries of gauge theories allow for specialized error tolerant/corrected calculations?[56–61]*
- *Can integrating out or particular discretizations of quantum fields reduce resources? [16, 19, 49, 62–64]*
- *How can efficient renormalization of quantum calculations be performed?[23, 44]*
- *Are Symanzik-like improvements of practical use to near-term quantum simulations?[65–68]*

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