Exploring Digitizations of Quantum Fields for Quantum Devices

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In this LOI we undertake to enumerate promising digitization schemes for quantum fields that could allow near-term calculations on quantum devices. Further we discuss the outstanding questions that must be resolved in evaluating their potential, providing potential benchmarking on the way to practical quantum advantage in high energy physics.

I. INTRODUCTION

Large-scale quantum computing would be capable of solving problems of incredible interest for high energy physics [1]. Alas for the foreseeable future, hardware limitations will restrict the scope of problems approachable by quantum devices. To take advantage of these devices, efficient *digitizations* of quantum field theories (QFT) must be developed. By digitization, we mean the task of formulating, representing, and encoding QFT (choosing the basis) in ways useful for computational calculations. The conventional digitization of nonperturbative field theory used on classical computers – lattice field theory (LFT) – relies upon resources far beyond near-term quantum machines. In conventional LFT, fermionic fields are integrated out, leaving a nonlocal action. A direct application of this procedure to quantum computers would requiring high connectivity between qubits. For bosons, LFT represents the infinite-dimensional local Hilbert space by floating-point numbers. This is prohibitively expensive in the so-called noisy, intermediate-scale quantum (NISQ) era. Moreover, Hamiltonian formulations of QFT may prove useful alongside Lagrangian-based ones. In summary, we anticipate novel digitizations for QFTs may be required for quantum computation.

In this LOI, we undertake to enumerate a list a proposals that show promise at allowing near-term calculations of quantum theory on quantum devices. Further, we point out some outstanding questions that will need to be resolved and provide potential metrics to compare the different prescriptions. The obstacles to digitizing fermions and bosons (gauge or otherwise) are quite different.

For fermions, while their local Hilbert space is finite, enforcement of their anticommutation relations often require nonlocal mappings onto qubits [2–4]. Despite this, in low dimensions some Hamiltonians can be rendered local, although some observables may not. Another consideration is the doubling problem [5]. There are proposals which use gauge symmetry to eliminate fermions [6, 7].

There are two main issues with digitizing gauge bosons. First, the infinite-dimensional local Hilbert space must be truncated. Second, constraints from gauge redundancy must either be imposed or higher qubit counts are needed to represent unphysical states. Differences between the digitizations are in how they address these issues. Like existing methods in computational physics, we expect that no single method is perfect for all situations – thus our emphasis on studying and benchmarking as many as possible in these early stages.

II. DIGITIZATION SCHEMES

Casimir dynamics: SU(2) gluon dynamics has been demonstrated for a toy geometry (string of plaquettes) on quantum hardware by analytically solving Gauss's law at vertices [8], leaving only Casimir eigenvalues as dynamical variables. This approach lays somewhere between Kogut-Susskind LFT [9, 10] and LSH digitization (below).

Discrete Subgroups: Approximating continuous gauge groups by discrete subgroups reduces qubit costs [11–13]. This remnant gauge symmetry will simplify renormalization, in particular from gauge-violation. The relation to

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LFT also simplifies the theoretical analysis [14, 15] and developing algorithms [16]. While pointwise matchings to continuous groups breakdown near the continuum, modified actions [13, 15] or other matchings [17] may mitigate this.

Dual variables: Many physically relevant and interesting models in LFT have compact degrees of freedom (DOF). These are usually either discrete and closed, or continuous and periodic. This compactness means the Boltzmann weight from the lattice action admits a discrete (but possibly infinite) character expansion in terms of the irreducible representations of the symmetry group of the action. These irreducible representations are dual variables [18–20] in the Pontryagin sense [21]. This expansion integrates out the original variables while the original symmetries of the model are preserved. The new discrete DOF are conducive for encoding quantum states. Moreover, truncating the expansion allows for a systematic approximation of the local Hilbert space and thus controls the accuracy of simulations [22–26].

Light-Front: Hamitonian QFT need not be formulated on fixed time-slices. Another formulation, within additional kinematic symmetries quantizes on the light-cone $x^+ = t + z$ [27]. A digitization was proposed using Hamiltonian derived on the light-cone [28] using a momentum basis truncated at a harmonic resolution.

Loop-String-Hadron (LSH): By coupling prepotentials [29–36] (Schwinger bosons) to fundamental fermions, one can construct a Loop-String-Hadron formalism [37], which expresses non-Abelian [SU(2)] dynamics in terms of strictly charge-conserving underlying operators. The ordinary non-Abelian Gauss law constraints are made automatic, though auxiliary U(1) constraints are introduced and imposed. Quantum circuits have been constructed to detect unphysical LSH basis states [38]. The LSH Hamiltonian is naturally expressed as a sum of one-sparse terms, which could benefit time evolution subroutines. The framework is amenable to use on either analog or digital devices.

Quantum Link Models: Quantum link models [39, 40] and their generalization D-theory [41, 42] digitize $U(N)$ and $SU(N)$ lattice field theories in the finite spin basis in extra dimensions while preserving gauge symmetry. This makes them potentially well-suited for quantum simulation [43]. Presently, quantum circuits for simulating the real-time evolution have been constructed for 1+1d and 2+1d $U(1)$ gauge theories [44, 45].

Qubit Regularization: Qubit regularization of a QFT refers to a quantum lattice Hamiltonian with a quantum critical point in a given universality class, such that the desired QFT emerges at long distances in the vicinity of the critical point. For example, recently, it has been shown that the O(3) nonlinear sigma model admits a qubit regularization with only two qubits per lattice [46, 47], and efficient algorithms for ground state preparation in this model have been developed [48]. Qubit regularizations of gauge theories have also been achieved in the context of quantum link models [39, 40] and D-theory [41, 42].

III. TOWARD QUANTITATIVE COMPARISONS

Listed are potential metrics which, as the schemes and hardware mature, can clarify the use cases for each of the above digitization schemes.

Basic resources: NISQ-era limitations on quantum resources must be considered. Digitizations with fewer qubits per DOF and shallow circuit depth are desired. Schemes that scale poorly for large DOF or are hardware-specific may prove useful for NISQ algorithms and early studies. Longer-term, error-correction may place a relative premium on efficient qubit use over other metrics. Initial results suggest the digitization-specific Hamiltonians vary greatly in their locality, sparseness, and two-qubit gate cost. These differences warrant further study like Ref. [49].

Robustness to Errors: Quantum noise will affect encoding differently. Near-term calculations on quantum hardware should elucidate these errors and could lead error-tolerant schemes. For example, errors can introduce gauge violation. This issue can be mitigated on the fly [50–55], or by using digitizations with only physical states [37, 56–59]. Additonally, this leakage can itself be leveraged as a metric of the device performance. How noise affects other symmetries needs further study.

Approach to Physical Point: Ultimately, the comparison that matters most is which digitizations produce results in the limit where all truncations and approximations have been removed – the physical point. Today, these final resources needed are largely unknown for all digitizations. Quantum calculations [60], analytical work [15, 61], and classical simulations [12, 13] can help address this, although the classical methods (e.g. Hamiltonian diagonalization or Monte Carlo) may be restricted due to exponential state-spaces or sign problems inherent to the schemes.

Benchmarks: An important comparison is through agreed-upon benchmarks. The earliest of these will be in 1+1d. While insightful, caution should be exercised because one can take advantage of simplifying features of 1+1d theories that do not persist in higher dimensions. For example, 2+1d QED offers another milestone, where renormalization and dimensionality complicate things. State preparation is generally hard [16, 62–69] and its demonstration would be another valuable type of test.

[1] R. P. Feynman, Simulating physics with computers, Int. J. Theor. Phys. **21**, 467 (1982).
[2] P. Jordan and E. P. Wigner, About the Pauli exclusion principle, Z. Phys. **47**, 631 (1928).

[3] S. Bravyi and A. Y. Kitaev, Fermionic quantum computation, Annals of Physics **298**, 210 (2002).
[4] Y.-A. Chen and A. Kapustin, Bosonization in three spatial dimensions and a 2-form gauge theory, Phys. Rev. B **100**, 245127 (2019), arXiv:1807.07081 [cond-mat.str-el].
[5] H. B. Nielsen and M. Ninomiya, No Go Theorem for Regularizing Chiral Fermions, Phys. Lett. B **105**, 219 (1981).
[6] E. Zohar and J. I. Cirac, Eliminating fermionic matter fields in lattice gauge theories, Phys. Rev. B **98**, 075119 (2018), arXiv:1805.05347 [quant-ph].
[7] E. Zohar and J. I. Cirac, Removing Staggered Fermionic Matter in $U(N)$ and $SU(N)$ Lattice Gauge Theories, Phys. Rev. D **99**, 114511 (2019), arXiv:1905.00652 [quant-ph].
[8] N. Klco, J. R. Stryker, and M. J. Savage, SU(2) non-Abelian gauge field theory in one dimension on digital quantum computers, Phys. Rev. D **101**, 074512 (2020), arXiv:1908.06935 [quant-ph].
[9] J. Kogut and L. Susskind, Hamiltonian formulation of wilson's lattice gauge theories, Phys. Rev. D **11**, 395 (1975).
[10] T. Byrnes and Y. Yamamoto, Simulating lattice gauge theories on a quantum computer, Phys. Rev. **A73**, 022328 (2006), arXiv:quant-ph/0510027 [quant-ph].
[11] M. Creutz, L. Jacobs, and C. Rebbi, Monte Carlo Study of Abelian Lattice Gauge Theories, Phys. Rev. **D20**, 1915 (1979); M. Creutz and M. Okawa, Generalized Actions in $Z(p)$ Lattice Gauge Theory, Nucl. Phys. **B220**, 149 (1983); G. Bhanot and C. Rebbi, Monte Carlo Simulations of Lattice Models With Finite Subgroups of SU(3) as Gauge Groups, Phys. Rev. **D24**, 3319 (1981); D. Petcher and D. H. Weingarten, Monte Carlo Calculations and a Model of the Phase Structure for Gauge Theories on Discrete Subgroups of SU(2), Phys. Rev. **D22**, 2465 (1980); G. Bhanot, SU(3) Lattice Gauge Theory in Four-dimensions With a Modified Wilson Action, Phys. Lett. **108B**, 337 (1982); D. H. Weingarten and D. N. Petcher, Monte Carlo Integration for Lattice Gauge Theories with Fermions, Phys. Lett. **99B**, 333 (1981); D. Weingarten, Monte Carlo Evaluation of Hadron Masses in Lattice Gauge Theories with Fermions, Phys. Lett. **109B**, 57 (1982), [,631(1981)].
[12] D. C. Hackett, K. Howe, C. Hughes, W. Jay, E. T. Neil, and J. N. Simone, Digitizing Gauge Fields: Lattice Monte Carlo Results for Future Quantum Computers, Phys. Rev. A **99**, 062341 (2019), arXiv:1811.03629 [quant-ph].
[13] A. Alexandru, P. F. Bedaque, S. Harmalkar, H. Lamm, S. Lawrence, and N. C. Warrington (NuQS), Gluon field digitization for quantum computers, Phys.Rev.D **100**, 114501 (2019), arXiv:1906.11213 [hep-lat].
[14] E. H. Fradkin and S. H. Shenker, Phase Diagrams of Lattice Gauge Theories with Higgs Fields, Phys. Rev. D **19**, 3682 (1979).
[15] Y. Ji, H. Lamm, and S. Zhu (NuQS), Gluon Field Digitization via Group Space Decimation for Quantum Computers, (2020), arXiv:2005.14221 [hep-lat].
[16] H. Lamm, S. Lawrence, and Y. Yamauchi (NuQS), General Methods for Digital Quantum Simulation of Gauge Theories, Phys. Rev. **D100**, 034518 (2019), arXiv:1903.08807 [hep-lat]; S. Harmalkar, H. Lamm, and S. Lawrence (NuQS), Quantum Simulation of Field Theories Without State Preparation, (2020), arXiv:2001.11490 [hep-lat].
[17] J. F. Haase, L. Dellantonio, A. Celi, D. Paulson, A. Kan, K. Jansen, and C. A. Muschik, A resource efficient approach for quantum and classical simulations of gauge theories in particle physics, (2020), arXiv:2006.14160 [quant-ph].
[18] R. Savit, Duality in field theory and statistical systems, Rev. Mod. Phys. **52**, 453 (1980).
[19] Y. Liu, Y. Meurice, M. P. Qin, J. Unmuth-Yockey, T. Xiang, Z. Y. Xie, J. F. Yu, and H. Zou, Exact blocking formulas for spin and gauge models, Phys. Rev. D **88**, 056005 (2013).
[20] D. B. Kaplan and J. R. Stryker, Gauss's Law, Duality, and the Hamiltonian Formulation of U(1) Lattice Gauge Theory, (2018), arXiv:1806.08797 [hep-lat].
[21] L. Pontryagin, *Topological Groups* (Princeton, 1939).
[22] J. Zhang, J. Unmuth-Yockey, J. Zeiher, A. Bazavov, S. W. Tsai, and Y. Meurice, Quantum simulation of the universal features of the Polyakov loop, Phys. Rev. Lett. **121**, 223201 (2018), arXiv:1803.11166 [hep-lat].
[23] J. Unmuth-Yockey, J. Zhang, A. Bazavov, Y. Meurice, and S.-W. Tsai, Universal features of the Abelian Polyakov loop in 1+1 dimensions, Phys. Rev. **D98**, 094511 (2018), arXiv:1807.09186 [hep-lat].
[24] A. Bazavov, S. Catterall, R. G. Jha, and J. Unmuth-Yockey, Tensor renormalization group study of the non-abelian higgs model in two dimensions, Phys. Rev. D **99**, 114507 (2019).
[25] A. Bazavov, Y. Meurice, S.-W. Tsai, J. Unmuth-Yockey, and J. Zhang, Gauge-invariant implementation of the Abelian Higgs model on optical lattices, Phys. Rev. **D92**, 076003 (2015), arXiv:1503.08354 [hep-lat].
[26] J. F. Unmuth-Yockey, Gauge-invariant rotor Hamiltonian from dual variables of 3D $U(1)$ gauge theory, Phys. Rev. D **99**, 074502 (2019), arXiv:1811.05884 [hep-lat].
[27] S. J. Brodsky, H.-C. Pauli, and S. S. Pinsky, Quantum chromodynamics and other field theories on the light cone, Phys. Rept. **301**, 299 (1998), arXiv:hep-ph/9705477.
[28] M. Kreshchuk, W. M. Kirby, G. Goldstein, H. Beauchemin, and P. J. Love, Quantum Simulation of Quantum Field Theory in the Light-Front Formulation, (2020), arXiv:2002.04016 [quant-ph].
[29] M. Mathur, Harmonic oscillator prepotentials in SU(2) lattice gauge theory, J. Phys. A **38**, 10015 (2005), arXiv:hep-lat/0403029.
[30] M. Mathur, Loop Approach to Lattice Gauge Theories, Nucl. Phys. B **779**, 32 (2007), arXiv:hep-lat/0702007.
[31] R. Anishetty and I. Raychowdhury, SU(2) lattice gauge theory: Local dynamics on nonintersecting electric flux loops, Phys. Rev. D **90**, 114503 (2014), arXiv:1408.6331 [hep-lat].
[32] I. Raychowdhury, Low energy spectrum of SU(2) lattice gauge theory: An alternate proposal via loop formulation, Eur. Phys. J. C **79**, 235 (2019), arXiv:1804.01304 [hep-lat].
[33] R. Anishetty, M. Mathur, and I. Raychowdhury, Irreducible SU(3) Schhwinger Bosons, J. Math. Phys. **50**, 053503 (2009), arXiv:0901.0644 [math-ph].
[34] R. Anishetty, M. Mathur, and I. Raychowdhury, Prepotential formulation of SU(3) lattice gauge theory, J. Phys. A **43**,

035403 (2010), arXiv:0909.2394 [hep-lat].
[35] M. Mathur, I. Raychowdhury, and R. Anishetty, SU(N) Irreducible Schwinger Bosons, J. Math. Phys. **51**, 093504 (2010), arXiv:1003.5487 [math-ph].
[36] I. Raychowdhury, *Prepotential Formulation of Lattice Gauge Theories*, Ph.D. thesis, Calcutta U. (2013).
[37] I. Raychowdhury and J. R. Stryker, Loop, String, and Hadron Dynamics in SU(2) Hamiltonian Lattice Gauge Theories, Phys. Rev. D **101**, 114502 (2020), arXiv:1912.06133 [hep-lat].
[38] I. Raychowdhury and J. R. Stryker, Solving Gauss's Law on Digital Quantum Computers with Loop-String-Hadron Digitization, (2018), arXiv:1812.07554 [hep-lat].
[39] S. Chandrasekharan and U. J. Wiese, Quantum link models: A Discrete approach to gauge theories, Nucl. Phys. **B492**, 455 (1997), arXiv:hep-lat/9609042 [hep-lat].
[40] R. Brower, S. Chandrasekharan, and U. J. Wiese, QCD as a quantum link model, Phys. Rev. **D60**, 094502 (1999), arXiv:hep-th/9704106 [hep-th].
[41] B. B. Beard, R. C. Brower, S. Chandrasekharan, D. Chen, A. Tsapalis, and U. J. Wiese, D-theory: Field theory via dimensional reduction of discrete variables, *Contents of LAT97 proceedings*, Nucl. Phys. Proc. Suppl. **63**, 775 (1998), [,775(1997)], arXiv:hep-lat/9709120 [hep-lat].
[42] R. Brower, S. Chandrasekharan, S. Riederer, and U.-J. Wiese, D-theory: field quantization by dimensional reduction of discrete variables, Nuclear Physics B **693**, 149 (2004).
[43] U.-J. Wiese, Towards Quantum Simulating QCD, *Proceedings, 24th International Conference on Ultra-Relativistic Nucleus-Nucleus Collisions (Quark Matter 2014): Darmstadt, Germany, May 19-24, 2014*, Nucl. Phys. **A931**, 246 (2014), arXiv:1409.7414 [hep-th].
[44] D. Luo, J. Shen, M. Highman, B. K. Clark, B. DeMarco, A. X. El-Khadra, and B. Gadway, A Framework for Simulating Gauge Theories with Dipolar Spin Systems, (2019), arXiv:1912.11488 [quant-ph].
[45] R. C. Brower, D. Berenstein, and H. Kawai, Lattice Gauge Theory for a Quantum Computer, PoS **LATTICE2019**, 112 (2019), arXiv:2002.10028 [hep-lat].
[46] H. Singh, Qubit $O(N)$ nonlinear sigma models, (2019), arXiv:1911.12353 [hep-lat].
[47] H. Singh and S. Chandrasekharan, Qubit regularization of the $O(3)$ sigma model, Phys. Rev. D **100**, 054505 (2019), arXiv:1905.13204 [hep-lat].
[48] A. J. Buser, T. Bhattacharya, L. Cincio, and R. Gupta, Quantum simulation of the qubit-regularized O(3)-sigma model, (2020), arXiv:2006.15746 [quant-ph].
[49] A. F. Shaw, P. Lougovski, J. R. Stryker, and N. Wiebe, Quantum Algorithms for Simulating the Lattice Schwinger Model 10.22331/q-2020-08-10-306 (2020), arXiv:2002.11146 [quant-ph].
[50] K. Stannigel, P. Hauke, D. Marcos, M. Hafezi, S. Diehl, M. Dalmonte, and P. Zoller, Constrained dynamics via the Zeno effect in quantum simulation: Implementing non-Abelian lattice gauge theories with cold atoms, Phys. Rev. Lett. **112**, 120406 (2014), arXiv:1308.0528 [quant-ph].
[51] J. R. Stryker, Oracles for Gauss's law on digital quantum computers, Phys. Rev. **A99**, 042301 (2019), arXiv:1812.01617 [quant-ph].
[52] J. C. Halimeh and P. Hauke, Reliability of lattice gauge theories, (2019), arXiv:2001.00024 [cond-mat.quant-gas].
[53] H. Lamm, S. Lawrence, and Y. Yamauchi, Suppressing coherent gauge drift in quantum simulations (2020), arXiv:2005.12688 [quant-ph].
[54] M. C. Tran, Y. Su, D. Carney, and J. M. Taylor, Faster Digital Quantum Simulation by Symmetry Protection, (2020), arXiv:2006.16248 [quant-ph].
[55] J. C. Halimeh, H. Lang, J. Mildenberger, Z. Jiang, and P. Hauke, Gauge-Symmetry Protection Using Single-Body Terms, (2020), arXiv:2007.00668 [quant-ph].
[56] J. B. Kogut, An Introduction to Lattice Gauge Theory and Spin Systems, Rev. Mod. Phys. **51**, 659 (1979).
[57] M. Mathur and T. Sreeraj, Lattice gauge theories and spin models, Physical Review D **94**, 10.1103/physrevd.94.085029 (2016).
[58] N. Klco, E. F. Dumitrescu, A. J. McCaskey, T. D. Morris, R. C. Pooser, M. Sanz, E. Solano, P. Lougovski, and M. J. Savage, Quantum-classical computation of Schwinger model dynamics using quantum computers, Phys. Rev. **A98**, 032331 (2018), arXiv:1803.03326 [quant-ph].
[59] Z. Davoudi, M. Hafezi, C. Monroe, G. Pagano, A. Seif, and A. Shaw, Towards analog quantum simulations of lattice gauge theories with trapped ions, Physical Review Research **2**, 10.1103/physrevresearch.2.023015 (2020).
[60] B. Chakraborty, M. Honda, T. Izubuchi, Y. Kikuchi, and A. Tomiya, Digital Quantum Simulation of the Schwinger Model with Topological Term via Adiabatic State Preparation, (2020), arXiv:2001.00485 [hep-lat].
[61] N. Klco and M. J. Savage, Digitization of Scalar Fields for NISQ-Era Quantum Computing, (2018), arXiv:1808.10378 [quant-ph].
[62] S. P. Jordan, K. S. M. Lee, and J. Preskill, Quantum Algorithms for Quantum Field Theories, Science **336**, 1130 (2012), arXiv:1111.3633 [quant-ph].
[63] S. P. Jordan, K. S. M. Lee, and J. Preskill, Quantum Computation of Scattering in Scalar Quantum Field Theories, (2011), [Quant. Inf. Comput.14,1014(2014)], arXiv:1112.4833 [hep-th].
[64] S. P. Jordan, K. S. M. Lee, and J. Preskill, Quantum Algorithms for Fermionic Quantum Field Theories, (2014), arXiv:1404.7115 [hep-th].
[65] E. F. Dumitrescu, A. J. McCaskey, G. Hagen, G. R. Jansen, T. D. Morris, T. Papenbrock, R. C. Pooser, D. J. Dean, and P. Lougovski, Cloud Quantum Computing of an Atomic Nucleus, Phys. Rev. Lett. **120**, 210501 (2018), arXiv:1801.03897 [quant-ph].

[66] A. Peruzzo, J. McClean, P. Shadbolt, M.-H. Yung, X.-Q. Zhou, P. J. Love, A. Aspuru-Guzik, and J. L. O'brien, A variational eigenvalue solver on a photonic quantum processor, Nature communications **5**, 4213 (2014).

[67] A. Roggero, A. C. Y. Li, J. Carlson, R. Gupta, and G. N. Perdue, Quantum computing for neutrino-nucleus scattering (2019), arXiv:1911.06368 [quant-ph].

[68] D. Lee, J. Bonitati, G. Given, C. Hicks, N. Li, B.-N. Lu, A. Rai, A. Sarkar, and J. Watkins, Projected cooling algorithm for quantum computation, Physics Letters B **807**, 135536 (2020).

[69] E. J. Gustafson, Projective cooling for the transverse ising model, Physical Review D **101**, 10.1103/physrevd.101.071504 (2020).