Snowmass 2021 - Letter of Interest

Field theories on a quantum computer

TF Topical Groups: (check all that apply □/■)

■ (TF10) Quantum Information Science

Other Topical Groups:
CompF6

Contact Information:
Tanmoy Bhattacharya (Los Alamos National Laboratory) [tanmoy@lanl.gov]

Authors:
Tanmoy Bhattacharya (Los Alamos National Laboratory) [tanmoy@lanl.gov]
Alex Buser (Caltech) [alexbuser@caltech.edu]
Shailesh Chandrasekharan (Duke University) [sch@phy.duke.edu]
Hersh Singh (Duke University) [hersh@phy.duke.edu]
Rajan Gupta (Los Alamos National Laboratory) [rajan@lanl.gov]

Abstract:
Quantum simulation of quantum field theories is likely to provide access to quantities that are not efficiently calculable by classical simulations. These include real time dynamics and simulation of field theories plagued by sign problems when they are formulated in Euclidean time. The nonperturbative definition of field theories in more than two dimensions usually proceeds by regularizing the theory on a spatial lattice, and tuning the parameters to a quantum critical point. To numerically simulate such a system on a finite quantum computer, the Hilbert space at each lattice site or link needs to be further truncated to a finite dimensional subspace. Because of universality, however, the truncation of the dimensionality of the Hilbert space at each site becomes irrelevant as one approaches the critical point, but different truncations might differ in their efficiency and their ability to be simulated on near-term quantum devices. Since the first quantum computers large enough to test algorithms for simulating field theories are likely to make their appearance in the next five to ten years, studying the implementation of algorithms to create, evolve, and measure interesting field theory states—and charting out the questions where quantum computation provides an advantage—should be studied now.

Motivation and Physics Goals

Efficiently simulating quantum field theories is a main challenge of high energy physics. When perturbation theory fails, the only first-principles method that is implementable on a classical computer beyond two spacetime dimensions involves taking the continuum limit of a discretized path integral evaluated by importance sampling. Often, however, the relevant measure is not positive when the paths are expressed in any known set of classical variables, and the integral becomes exponentially hard to compute since the sum over an exponentially large number of paths becomes necessary. Hence, novel computational paradigms are highly desirable.

Quantum computers can, in principle, overcome the challenges that we currently face on a classical computer, since one quantum system can efficiently simulate a different quantum system using similar resources. However, one has to face three challenges before quantum computers can become useful for high energy physics. The first challenge is related to the fundamental question of whether traditional field theories with bosonic degrees of freedom, including gauge fields, which are formulated with an infinite dimensional local Hilbert space, can be formulated with a finite number of qubits so as to be able to tractable on a quantum computer. The second challenge is to design quantum algorithms to probe interesting physical properties of such "qubit" field theories which can be implemented on quantum devices. In particular, one will need algorithms for expectation values of physical observables and correlations in thermal equilibrium as well as time-dependent properties of QFTs such as creating, evolving, and measuring interesting states in the theory. While quantities that are out of thermal equilibrium may also be accessible, it may be possible to design novel hybrid algorithms that improve over currently used classical algorithms for equilibrium observables. The third challenge comes from the fact that forthcoming quantum computers will have only limited coherence and will be unable to implement quantum circuits of arbitrary depth. Thus, it is crucial to formulate the field theory in a language amenable to a small, noisy quantum device.

Qubit Regularizations and Quantum Critical Points

Local quantum field theories of interest in high energy physics are naturally defined using an infinite-dimensional Hilbert space. The infinities come from two sources. The first source is the infinite number of spatial points even within the small continuum region. A natural way to regularize this ultra-violet (UV) infinity, proposed by Wilson almost 50 years ago, is to consider the low-energy sector of a finite-dimensional quantum system defined on a spatial lattice. QFTs emerge in such low energy sectors of lattice quantum systems only near the vicinity of quantum critical points, whose existence then becomes an important criteria to define QFTs non-perturbatively. The second source of infinity is the need for an infinite local Hilbert space even on the finite lattice, as is implied from the well known canonical commutation relation

$$[\phi(x), \pi(y)] = i\delta_{x,y}$$

where x and y are spatial discrete lattice sites. There are no finite dimensional realizations of this commutation relation. Yet, free quantum field theories and the physics of asymptotic freedom rely heavily on this relation. Any truncation of the local Hilbert space necessary for implementing a quantum field theory on a quantum computer will necessarily destroy the relation, making fine tuning a necessary step before free quantum field theories can emerge, especially in the UV.

The first problem then is the one of finding an appropriate quantum theory on a lattice with both a finite dimensional local Hilbert space and a quantum critical point in the desired universality class. We define this as a *qubit regularization*. Once this is accomplished, one needs a way of creating the states of interest in this theory: e.g., states whose long distance properties at the quantum critical point match those of few particle ‘in’ states or of thermal states in the field theory. These then need to undergo quantum evolution using a quantum algorithm with short circuit depth to be implementable on near-term quantum devices. Finally, the quantities of interest have to be efficiently measured, e.g., by calculating expectation values or by projecting onto few particle ‘out’ states.

Recent work has shown that it is likely that the $O(3)$ sigma model appears as the long-distance description of a lattice Hamiltonian constructed with two qubits per lattice site when tuned to the critical point of the theory[5]. This formulation can also be extended to $O(N)$ sigma models for arbitrary N, with a small local Hilbert space[4]. New quantum critical points in gauge theories have also been shown to exist in highly truncated theories[3]. While an approach based on qudit-registers, as in traditional proposals for simulating field theories, can be efficiently encoded using two-qubit interactions[2], we advocate for this simpler approach where the microscopic lattice Hamiltonian is directly realized as a local interaction between the logical degrees of freedom of a quantum device. We anticipate that such an approach, which frames universality as an issue of central importance, will not only aid in the development of near-term experimental applications, but may lead to a reduction in resource requirements. The ground state, or vacuum, of this two-qubit theory can be efficiently prepared with a low depth quantum circuit[1]. General algorithms also exist for preparing single and two particle states in free field theories. Some work has also been carried out in efficient preparation of thermal states in field theories. The next few years will see rapidly developing techniques for carrying out such calculations, both on quantum devices, and with hybrid quantum-classical algorithms.

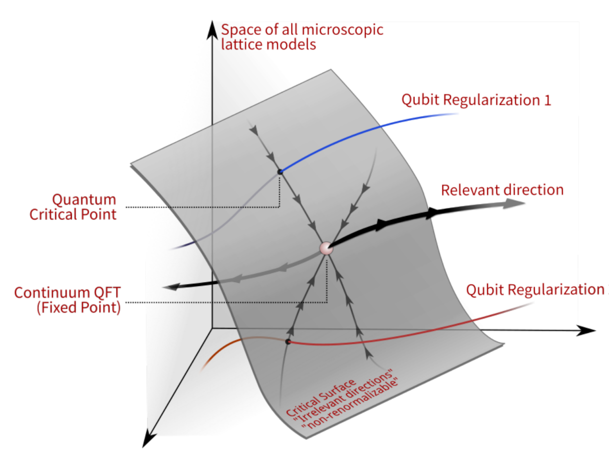

Figure 1: Schematic of how qubit regularization fits into the usual picture of Wilson’s renormalization group ideas. The two lines shown as Qubit Regularization 1 and Qubit Regularization 2 show a set of qubit Hamiltonians where one parameter is varied. These are distinct from RG flow lines, which are shown with arrows.

References

[1] A. J. Buser, T. Bhattacharya, L. Cincio, and R. Gupta. Quantum simulation of the qubit-regularized O(3)-sigma model. *arXiv:2006.15746*, (2020).

[2] T. Byrnes and Y. Yamamoto. Simulating lattice gauge theories on a quantum computer. *Phys. Rev. A*, 73:022328, Feb 2006.

[3] J. Frank, E. Huffman, and S. Chandrasekharan. Emergence of Gauss' law in a Z_2 lattice gauge theory in 1 + 1 dimensions. *Phys. Lett. B*, 806:135484, (2020).

[4] H. Singh. Qubit $O(N)$ nonlinear sigma models. 11 2019.

[5] H. Singh and S. Chandrasekharan. Qubit regularization of the $O(3)$ sigma model. *Phys. Rev. D*, 100(5):054505, (2019).