2020 snowmass Letter of Interest: TMD PDF in large-momentum effective theory

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The transverse-momentum-dependent (TMD) parton distribution functions (TMDPDFs) are a natural generalization of the collinear PDFs to include both longitudinal and transverse momentum of partons. They are important in understanding the experimental processes where the transverse momenta of final state particles are measured, such as the Drell-Yan process [1, 2] at small transverse momentum of the observed lepton pair, or the semi-inclusive deep-inelastic scattering process at small transverse momentum of the observed hadron [3]. They are also important by themselves for their crucial role in describing hadron structures. With them, one can simultaneously study the longitudinal x-dependencies and the transverse $\vec{k}_\perp$-dependencies inside a fast-moving hadron. Therefore, compared to collinear PDFs, the TMDPDFs represent a further step towards a more complete and refined 3D description (or tomography) of the hadron structure, which is a major physical goal of the EIC program. From the field theoretical point of view, the TMD quantities are much more complicated than the standard collinear PDFs due to the presence of light-cone divergences [3–6]. The non-triviality of the light-cone limit is perfectly demonstrated in the TMD factorization formalism. To have a better understanding of this limit, a good knowledge of TMD factorization and TMD PDFs is also necessary.

Traditionally, our knowledge on TMDPDFs mainly comes from fitting to the experimental data. This is, however, rather primitive to the paucity of data. Although the future EIC will make up the gap and produce more data for TMD measurements, it is still important to develop first-principle methods for the determination of nonperturbative TMDPDFs, which can serve as a test or provide useful inputs to constrain the global fits. The large momentum effective theory(LaMET) [7–9] provides a systematic way to extract TMDPDFs from the lattice calculations. The LaMET approach to TMDPDF starts with the lattice calculable quasi-TMDPDFs, then match them to TMDPDFs with the help of lattice-calculable soft functions and perturbative matching coefficients. The required soft function that serve this purpose has been investigated by the community for many years [10–12], only recently [13, 14] we have found a formalism that allows direct simulation on lattice, and a first calculation on lattice has already appeared [15]. This opens up the future possibility of obtaining the TMDPDFs from lattice calculation by combing the calculation of soft function and that for quasi-TMDPDFs. Moreover, the LaMET formulation of TMDPDFs requires a good understanding of many subtle points of the TMD factorization formalism, in particular, the property of off-light-cone regulators and the relation to the on-light-cone one. A better understanding of these issues will greatly increase our understanding of the light-cone limit. Therefore, the LaMET formulation of the TMD problem will also be important in the field-theoretical side.

In summary, the following topics are of interests to the lattice, nuclear and theory community:

- To understand the properties (such as analyticity) of the off-light-cone soft functions and their relation to the on-light-cone one better.
- Provide the rigorous factorization proof of the quasi-TMDPDFs.
- Investigate possible Euclidean formulation for other type of soft functions, such as the threshold soft function, energy-energy correlators, etc.
- Investigate the high-twist effect to the quasi-TMDPDFs.
- To understand the renormalization property of the staple-shaped gauge-links better, in particular, the general pattern of operator mixing to all orders in lattice perturbation theory.
- To carry out high-loop calculations for quasi-TMDPDFs and off-light-cone soft functions. Develop methods suitable for off-light-cone calculations.

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