# Light-front wavefunction from lattice QCD through large-momentum effective theory

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Light-front quantization (LFQ) or formalism is a natural language for parton physics in which partons are made manifest at all stages of calculations. It favors a Hamiltonian approach to QCD like for a non-relativistic quantum mechanical system to obtain wave functions for the QCD bound states [1]. It is based on the intuition that the vacuum in the LF is trivial, as such, there is a clear separation of partons from the vacuum. As a result, we can expand any hadron state in a complete set of free Fock states. The expansion coefficients are called the Light-Front wave function amplitudes(LFWF amplitudes). They can, in principle, be used to calculate all the partonic densities and correlations functions. After proper regularization and renormalization, the LFWF amplitudes become physical observables and play important role in exclusive process such as $B$ decays [2, 3] or to describe hadronic form-factors such as the pion electro-magnetic form-factor [4, 5], the proton form-factor [6, 7]. Therefore, a practical realization of LFQ program clearly would be a big step forward in understanding the fundamental structure of the proton. The real implementation of the LFQ is, however, much more complicated than its conceptual simplicity due to the famous zero-mode problem which requires careful regularization and renormalization. For simple models in lower dimensions the analysis is possible [8], but for gauge-theories in 4-dimensions the problem caused by UV divergence as well as the light-cone divergence is so complicate such that a direct systematic solution of 4-d gauge theory in LFQ is impractical [9].

The large momentum effective theory(LaMET) [10–12] offers an alternate route to calculate these WF amplitudes. Instead of trying to solve these amplitudes from Hamiltonian dynamics, LaMET made the observation that the LFWF amplitudes can be rewritten as matrix elements of light-cone correlators, the same type as those in the TMDPDFs. Therefore, the LaMET method applies to them as well, which allows effectively obtain the results of LFQ through instant quantization in a large momentum frame [12].

Moreover, the implementation of the LaMET approach to LFWF amplitude is important for the purpose of obtaining the off-light-cone TMD soft functions that appears in the LaMET formulation of TMDPDFs. It has been shown [13] that the off-light-cone soft functions can be obtained by combining a light-cone form-factor at large momentum transfer and their leading component of LFWF amplitude. When the transverse momentum is integrated over, these LFWF amplitudes reduces to the light-front distribution amplitudes (LFDA), whose lattice calculation has been studied in Ref. [14–16] and carried in Refs. [17–19]. Moreover, the LFWF amplitudes can also be used to extract the nonperturbative rapidity anomalous dimension, or Collins-Soper kernel, which is universal among LFWFs and TMDPDFs, as has been carried out in a recent lattice calculation in Ref. [20]. A first calculation of the soft function from LFWF amplitudes on lattice has already appeared [20] without implementation of one-loop matching and exploring the full $z$-dependence. The next step is naturally to calculate and renormalize the LFWF amplitude to include the full $z$-dependence and one-loop matching to make the first lattice prediction of the physical leading LFWF amplitude of pion.

As such, the following topics are of interests to the lattice, nuclear and theory community:

- To understand the lattice renormalization of the staple-shaped gauge-links that defining the quasi-LFWF amplitudes better, in particular, the general pattern of operator mixing beyond leading order in lattice perturbation theory.
- To calculate the full $z$ and $x$ dependencies of the quasi-LFWFs and combine them with the already obtained off-light-cone soft functions to produce the physical LFWF amplitudes that can be compared with experiment. For this purpose, the one-loop matching in appropriate renormalization scheme requires further calculation.
- Formulate the leading nucleon wave functions in LaMET, and calculate one-loop matching.
- Formulate the heavy-meson such as B-meson TMD-wave functions in LaMET, and calculate one-loop matching.
- Perform two-loop calculation of the quasi-LFWFs for light-meson and other hadrons.
- There are many theoretical problems related to the light-cone limit that can be explored in our framework. These have been explained in the TMDPDF interests paper. Many other applications besides the calculation of TMDs would profit from progress made along these lines, e.g., the calculation of DAs. However, these other applications are beyond the scope of this letter.

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