Lorentz and CPT Breakdown as BSM Physics and its EFT Description

Snowmass21 Letter of Interest submitted to:

Theory Frontier

Topical Group TF02: EFT Techniques

Topical Group TF08: BSM Model Building

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Lorentz and CPT symmetry represent intimately related foundational aspects of physics, and in particular they determine many basic features of spacetime. However, the quest for a quantum description of the dynamics of spacetime is widely believed to involve modifications to this existing theoretical structure, which become sizable at the Planck scale. Departures from Lorentz and CPT invariance have been established to be viable candidate signatures in a variety of BSM model-building efforts in this context. Regardless of the physical details of such efforts, all perturbative Lorentz and CPT violations emerging at presently attainable energy scales are expected to be governed by an EFT that contains the Standard Model and General Relativity as limiting cases. This general approach to the subject has yielded the Standard-Model Extension. As the basis for over 300 modern Lorentz and CPT tests, this EFT framework has had tremendous experimental impact with numerous measurements achieving Planck sensitivity. This Letter describes the vitality of theoretical research in this field and argues that intensified activities are expected in the coming decade.

Motivation.—At a fundamental level, all currently established laws of nature rely on a description in terms of a four-dimensional classical Riemannian manifold: such a spacetime both provides the arena for physical events and responds dynamically to these events. However, reconciling quantum mechanics with gravity in a self-consistent way is expected to involve modifications to this spacetime structure at the Planck length. For instance, spacetime dimensionality, its topological structure, its geometrical underpinnings, and its symmetries continue to be challenged by various theoretical approaches to BSM physics. In particular, Lorentz and CPT invariance are among the most scrutinized principles in physics, and to date no compelling experimental evidence for departures from these symmetries exists. Yet there is a long history as well as ongoing and current interest in studying departures from these symmetries. Early efforts proposed CPT violation in quantum gravity [1–3], emergent Lorentz symmetry [4], and spontaneous breaking of Lorentz symmetry and CPT invariance in string theory [5, 6]. Over the following three decades, numerous model-building efforts involving Lorentz and CPT violation have appeared, including in string [7–9] and non-string [10–14] approaches to quantum gravity, nontrivial spacetime topology [15], noncommutative field theory [16–21], emergent symmetries [22–27], multiverses [28], brane-world scenarios [29–31], cosmologically varying scalars [32, 33], nonlocal theories [34, 35], and in quantum and conformal field theories [36–43].

Lorentz and CPT symmetry underpins many key features in numerous physical systems across all energy scales. This fact, together with the availability of ultrahigh-precision measurement techniques, places such putative Planck-scale effects potentially within reach of present-day experimentation. The realization of a correspondingly comprehensive program for the experimental study of these symmetries calls for a general theoretical test framework. Such a framework needs to capture the multitude of observable signatures emerging from general Lorentz and CPT breakdown in underlying physics at presently attainable energies, and it should permit the identification, interpretation, and comparison of tests in the broadest range of physical systems. This strongly suggests a general perturbative EFT expansion about established physics, i.e., the construction and classification of small Lorentz- and CPT-violating corrections to the Standard-Model (SM) and Einstein–Hilbert Lagrangians.

Such a program, called the Standard-Model Extension (SME), was initiated over two decades ago [44–46]. Since then, the SME has been the focus of numerous studies confirming its theoretical viability, seeking conceptual insights into Lorentz and CPT violation, and extracting phenomenological predictions. At the same time, this EFT framework has provided the basis for over 300 Lorentz and CPT tests with photons and other gauge bosons, electrons, neutrons, protons, muons, neutrinos, quarks, the Higgs boson, and gravitational physics [47]. This volume of experimental efforts is truly astonishing and poised for leaps in sensitivity in the coming decade. These tests are spread across all frontiers in this Snowmass exercise that involve experimental searches for new physics, further illustrating their current status and future prospects in particle physics. The present Letter focuses on theoretical aspects in the field extrapolating past achievements to forecast sustained activity in the coming decade.

Classification of Departures from Lorentz and CPT Symmetry.—The construction of the SME represents a BSM model-building effort based on the EFT paradigm of parametrizing all interactions compatible with the symmetry properties of the presumed underlying physics. A key goal therefore is the classification and enumeration of as wide a range of departures from Lorentz and CPT invariance as possible. As a result of these EFT underpinnings, an obvious classification criterion are the mass dimensions of the operators that govern the various Lorentz- and CPT-violating corrections, which simultaneously provides a natural hierarchy for these terms. Such corrections can be further distinguished by, e.g., their C, P, and T properties, and whether they primarily affect particle propagation or interactions. During the last two decades, this classification program has picked up pace: all relevant and marginal operators are classified [44, 46], and many of their experimental signatures are known. At higher mass dimensions, quadratic contributions, with primary effects on particle propagation have been enumerated as well [48–51]. General gauge interactions in QED, QCD, and gravity have recently also been catalogued [52, 53]. A complete theoretical classification is the ultimate goal in this line of research and is expected to be achievable in the coming decade.

Model Building and the SME.—In addition to being itself a model-building effort, the SME also provides a framework for the construction of physics models for various purposes. These include models involving localized BSM background fields mimicking departures from Lorentz and CPT invariance, toy models addressing issues related to the origins of such effects, as well as those that restrict the pattern of Lorentz and CPT breakdown due to additional imposed constraints. Such efforts, aimed at gaining theoretical insight into the conceptual underpinnings of Lorentz and CPT breaking, fall squarely into the Theory Frontier. For example, imposing supersymmetry eliminates some SME contributions [54–56], as expected in an EFT construction. Other findings have shown that situations involving general torsion and nonmetricity backgrounds share their mathematical description with certain SME physics, a result also of interest for experimental studies [57–60]. Certain condensed-matter systems have lattice backgrounds, which exhibit many similarities with SME coefficients. This fact can be employed to gain insight into mechanisms generating Lorentz breakdown. A well-known example in this context is nondynamical axion electrodynamics [61], which describes the electromagnetic response of topological insulators [62] and Weyl semimetals [63]. A foreseen systematic approach in this direction will make available further condensed-matter tools and insights for particle physics. Some of these systems involve strong-coupling phenomena, which can be described using holographic tools [64, 65]. A related aim is therefore to investigate non-Lorentz invariant holographic correspondences [37, 38, 40]. The gravitational sector of the SME also allows cross-pollination with model-building in gravitational physics. Various extensions to General Relativity violate Lorentz symmetry [66, 67]. The SME is not only designed to capture their phenomenology; it also inspires new gravitational models [68]. Another set of studies has produced models elucidating various aspects of Lorentz violation including the degree of ubiquitousness of the interactions that lead to a spontaneous breakdown, maintenance of stability under renormalization-group flow, and the nature of the associated Nambu–Goldstone bosons. The results of these studies indicate, e.g., that suitable nonpolynomial potentials generically trigger spontaneous Lorentz breakdown, they provide alternative mechanisms protecting the masslessness of photons and gravitons, and show that unitarity can be maintained [35, 69–75, 77]. These investigations have merely shone spotlights on the numerous interesting topics to be explored in this context, and future BSM model building within the SME framework will undoubtedly continue to offer bottom-up theoretical insights into potential Lorentz and CPT violations.

EFT Methods for SME Applications.—EFT techniques are not only confined to the construction of the SME itself. A second layer of EFT tools has proved to be immensely valuable for SME physics involving the strong interaction: paralleling the ordinary SM, in which many aspects of hadronic physics can be addressed with chiral perturbation theory [78–80], an adaptation of these methods to the SME permits the interpretation of Lorentz tests with hadrons in terms of coefficients for their elementary quark and gluon constituents. Theoretical SME efforts along these lines, which are also based on the usual reasoning of approximate chiral symmetry for light quarks, have recently been instigated [81–84]. Initial results have led to dramatic improvements in extracting constraints from existing measurements, sometimes by more than ten orders of magnitude. These activities have set the stage for comprehensive future explorations of Lorentz and CPT violation in QCD including analyses of higher-spin mesons and baryons, as well as the interactions between different hadronic species.

In the usual SM, strong-interaction physics can also be extracted with other methods overlapping EFT that permit focusing on subprocesses that are amenable to a perturbative study. Since the SME framework is inherently perturbative, it stands to reason that an analogous, expanded treatment within the SME could yield relations between Lorentz-violating quark coefficients and observables [85]. For example, recent theoretical studies of high-energy hadronic processes within the SME have demonstrated factorization of the hadronic tensor at leading order in electroweak interactions for deep inelastic scattering and for the Drell–Yan process, and the equivalent parton-model description has been derived [86]. Sensitivity estimates for SME coefficients at HERA, the LHC, the LHeC, the FCC-eh, and the future US-based EIC have been obtained [86, 87]. These results are set to spawn future theoretical studies of a multitude of related processes, including charged-current, polarized lepton–hadron, and hadron–hadron interactions, as well as investigations of higher-order effects, such as QCD corrections.

It should not come as a surprise that many SME studies heavily draw on modifications of EFT methods commonly employed in the SM and other field theories. These include renormalization analyses [41, 88–95], investigations of radiative effects and issues related to Chern–Simons modifications [96–103], studies of the asymptotic Hilbert space [104–107], and the adaptation of tools for phenomenological explorations like cross sections [108–116]. Theoretical efforts along these lines are bound to extend well into the future, as they both solidify the theoretical basis of the SME and expand the array of physical systems with theoretically established suitability for Lorentz tests. For instance, as an EFT the SME is usually understood to be the experimentally relevant model already incorporating all quantum corrections without restrictive assumptions on the underlying bare Lorentz violation. But imposing mild assumptions together with renormalization-group studies may reveal relations between the various SME coefficients potentially leading to a substantial reduction in SME parameter space. Intriguing phenomenological examples for future efforts in this context include generalizations of heavy-quark effective theory and strong-field gravity.

Finsler Geometry.—Theoretical studies have revealed that an underlying Riemannian spacetime structure places severe constraints on possible departures from Lorentz and CPT invariance [46]. Although these constraints can be evaded in special situations [75, 117, 118], such as when the breakdown is spontaneous [46], it seems sensible to contemplate a more general set of underlying symmetry-violation sources. The tremendous success and conceptual appeal of a geometric description of spacetime suggests to broaden the class of acceptable manifold structures. In this context, Finsler geometry [119] has recently attracted considerable interest: it contains Riemannian geometry as a limiting case while inherently incorporating spacetime anisotropies. In particular, it was uncovered that wave-packet evolution of SME quantum particles follows Finsler geodesics [120]. The development of this mathematical tool for deployment in spacetime-symmetry research is expected to become a major component of theoretical research in this field [121–123]. At the same time, such results influence mathematical research, as they may help classify Finsler spaces via the classification of Lorentz violations in the SME [124].

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