Lattice field theory for conformal systems and beyond
Letter of Interest for TF03+TF05+CompF2

Andrea A. Carosso,[a,b] Anna Hasenfratz,[a*] Ethan T. Neil,[a] Claudio Rebbi,[c]
Enrico Rinaldi,[e,f] David Schaich,[d] Benjamin Svetitsky[g] and Oliver Witzel [a**]

August 31, 2020

Quantum field theories of gauge–fermion systems are a fundamental concept in theoretical physics. Most prominently quantum chromodynamics (QCD) plays a central role in the Standard Model of elementary particle physics and is crucial to understand e.g. the stability of matter. More generally, the infrared properties of gauge-fermion systems can vary drastically depending on the gauge group, the number of colors (N_c), and the number of fermions (N_f) in different gauge representations. Spontaneous chiral symmetry breaking and confinement as in QCD can give way to infrared conformality or possibly even asymptotic safety as the number of fermion degrees of freedom increases. Even though chirally broken systems like QCD are defined on the perturbative Gaussian fixed point (GFP), many of their infrared properties can only be explored using inherently nonperturbative methods, like lattice field theory (LFT) and lattice QCD.

As the number of fermion degrees of freedom increases, a new conformal infrared fixed point (IRFP) arises in addition to the GFP. The most interesting conformal fixed points are at strong coupling, but in spite of significant effort in both numerical and analytical calculations, not much is known about the mechanism of the opening of the conformal window. While the possibility that the IRFP arises together with a nonperturbative ultraviolet fixed point (UVFP) is appealing, there is no evidence yet for this phenomenon in 4-dimensional gauge–fermion systems [1–4]. For many systems even the critical number of flavors where the conformal window opens is still controversial. For example for SU(3) gauge theories with fundamental fermions critical values of N_f are named in the range $6 < N_f < 13$ [5–19]. Knowing the exact onset of the conformal window is relevant for composite Higgs models [20–29] which require near-conformal dynamics. Models of a composite Higgs boson, as well as a partially composite top quark, can be based on other gauge groups [23, 30], with an assortment of fermions in various representations of the gauge group. The determination of fixed point couplings, anomalous dimensions, and particle spectra is a wide area of research that requires lattice computation. Likewise, knowledge of the anomalous dimensions of composite operators like the scalar, tensor or baryon, is essential for composite models describing physics beyond the Standard Model (BSM). Lattice calculations can predict anomalous dimensions [6, 31–34] with new techniques under active development [35, 36].

For an even larger number of fermion degrees of freedom the gauge coupling at the GFP becomes irrelevant and asymptotic freedom is lost. These infrared free systems may be trivial or a new, nonperturbative/non-trivial UVFP might emerge. Large-N_f considerations predict such a UVFP [37], but so far numerical calculations have not been able to verify this prediction [38, 39]. This is an exciting direction where numerical calculations have just started.

Simulation techniques developed for lattice QCD studies are readily applicable to investigate many of the nonperturbative properties of the tantalizing near-conformal, conformal, and infrared-free sys-

---

[*] anna.hasenfratz@colorado.edu
[**] oliver.witzel@colorado.edu

tems described above. There is a fruitful exchange of ideas between lattice QCD and lattice BSM studies.

Lattice practitioners have been investigating near-conformal and conformal systems in the last decade, generalizing well-known methods like finite size scaling or the scaling of the spectrum. A promising approach to identify the phase and fixed point structure is to study the running coupling and the renormalization group (RG) β function. The finite volume step scaling function has been determined with high precision in 2- and 3-flavor QCD. A modified approach that might improve the precision even further has been proposed recently [40]. The calculation in near-conformal systems is much more difficult. Studies with $N_f = 10$ [10, 15, 41] and 12 [10, 13] fundamental flavors suggest that those systems are conformal, whereas $N_f = 8$ appears to be chirally broken [42–44]. A new method that calculates the continuous β function [4, 45] is a promising alternative which due to different systematic uncertainties provides a new perspective. Taking advantage of new methods and an improved understanding of numerical lattice artifacts [46], we aim to systematically study systems with $N_f = 4, 6, 8$ flavors. Combined with reanalyzing existing 10 and 12-flavor data, we expect to enhance our understanding of the opening of the conformal window. Our work will complement perturbative investigations of the RG β function [17, 18, 47–49] which so far have been pushed to 5-loop order in the $\overline{\text{MS}}$ scheme. However, at strong coupling perturbation theory may not be reliable and poor convergence in the perturbative series is observed for values of N_f considered to be close to the onset of the conformal window. The conformal bootstrap [50] is another powerful approach to study conformal QFTs and mutual exchanges will be beneficial for both groups.

If asymptotically safe systems exist, they would be very exciting for pure theoretical reasons. They could also be relevant in certain models of gravity [51]. Whether for larger N_f the infrared free range above the conformal window is followed by an asymptotically safe region, has only recently moved into the focus of numerical investigations and such projects are still in their early stages. A further interesting avenue to explore is the hypothesis of complex fixed points and the connection to Nambu – Jona-Lasinio type models [1, 2]. Such studies most likely require probing gauge–fermion systems with an added four-fermion interaction term. While the additional term complicates the parameter space, there is no numerical difficulty in carrying out such studies [52, 53].

Non-perturbative lattice field theory methods are also widely applicable to analyze broader classes of vector-like gauge theories. These include super-conformal maximally supersymmetric Yang–Mills theory ($\mathcal{N} = 4$ SYM) [21, 54–56], as well as lower-dimensional systems like QED$_3$ where both the onset of conformal behavior and even the existence of a chirally broken regime are controversial [57–60]. Methods developed for the 4-dimensional gauge–fermion systems can be generalized e.g. to ϕ^4 scalar models in 2 and 3 dimensions. For example, attempts to determine the leading and subleading critical exponents in the 3-dimensional ϕ^4 model have been presented in [61, 62].

Affiliations

[a] Department of Physics, University of Colorado, Boulder, CO 80309, USA
[b] Department of Physics, The George Washington University, Washington, DC 20052, USA
[c] Department of Physics and Center for Computational Science, Boston University, Boston, MA 02215, USA
[d] Department of Mathematical Sciences, University of Liverpool, Liverpool L69 7ZL, UK
[e] Arithmer Inc., R&D Headquarters, Minato, Tokyo 106-6040, Japan
[f] Interdisciplinary Theoretical and Mathematical Sciences Program (iTHEMS), RIKEN, 2-1 Hirosawa, Wako, Saitama 351-0198, Japan
[g] Raymond and Beverly Sackler School of Physics and Astronomy, Tel Aviv University, 96678 Tel Aviv, Israel

References

[1] D. B. Kaplan, J.-W. Lee, D. T. Son and M. A. Stephanov, *Conformality Lost*, *Phys. Rev.* **D80** (2009) 125005, [0905.4752].

[2] V. Gorbenko, S. Rychkov and B. Zan, *Walking, Weak first-order transitions, and Complex CFTs*, *JHEP* **10** (2018) 108, [1807.11512].

[3] A. Giuliani, V. Mastropietro and S. Rychkov, *Gentle introduction to rigorous Renormalization Group: a worked fermionic example*, 2008.04361.

[4] A. Hasenfratz and O. Witzel, *Continuous β function for the SU(3) gauge systems with two and twelve fundamental flavors*, 1911.11531.

[5] Z. Fodor, K. Holland, J. Kuti, D. Nogradi, C. Schroeder and C. H. Wong, *Conformal finite size scaling of twelve fermion flavors*, *PoS* **Lattice 2012** (2012) 279, [1211.4238].

[6] A. Cheng, A. Hasenfratz, G. Petropoulos and D. Schaich, *Scale-dependent mass anomalous dimension from Dirac eigenmodes*, *JHEP* **1307** (2013) 061, [1301.1355].

[7] K. I. Ishikawa, Y. Iwasaki, Y. Nakayama and T. Yoshie, *IR fixed points in $SU(3)$ gauge Theories*, *Phys. Lett.* **B748** (2015) 289–294, [1503.02359].

[8] T. Nunes da Silva, E. Pallante and L. Robroek, *Conformal or Confining*, 1506.06396.

[9] A. Hasenfratz and D. Schaich, *Nonperturbative beta function of twelve-flavor SU(3) gauge theory*, *JHEP* **02** (2018) 132, [1610.10004].

[10] A. Hasenfratz, C. Rebbi and O. Witzel, *Nonperturbative determination of β functions for SU(3) gauge theories with 10 and 12 fundamental flavors using domain wall fermions*, *Phys. Lett.* **B798** (2019) 134937, [1710.11578].

[11] Z. Fodor, K. Holland, J. Kuti, D. Nogradi and C. H. Wong, *Extended investigation of the twelve-flavor β-function*, *Phys. Lett.* **B779** (2018) 230–236, [1710.09262].

[12] Z. Fodor, K. Holland, J. Kuti, D. Nogradi and C. H. Wong, *Is SU(3) gauge theory with 13 massless flavors conformal?*, *PoS* **LATTICE2018** (2018) 198, [1811.05024].

[13] A. Hasenfratz, C. Rebbi and O. Witzel, *Gradient flow step-scaling function for SU(3) with twelve flavors*, *Phys. Rev.* **D100** (2019) 114508, [1909.05842].

[14] Z. Fodor, K. Holland, J. Kuti, D. Nogradi and C. H. Wong, *Case studies of near-conformal β-functions*, 1912.07653.

[15] A. Hasenfratz, C. Rebbi and O. Witzel, *Gradient flow step-scaling function for SU(3) with ten fundamental flavors*, *Phys. Rev. D* **101** (2020) 114508, [2004.00754].

[16] T. A. Ryttov and R. Shrock, *Scheme-Independent Series Expansions at an Infrared Zero of the Beta Function in Asymptotically Free Gauge Theories*, *Phys. Rev.* **D94** (2016) 125005, [1610.00387].

[17] O. Antipin, A. Maiezza and J. C. Vasquez, *Resummation in QFT with Meijer G-functions*, *Nucl. Phys. B* **941** (2019) 72–90, [1807.05060].

[18] L. Di Pietro and M. Serone, *Looking through the QCD Conformal Window with Perturbation Theory*, 2003.01742.

[19] T. A. Ryttov and R. Shrock, *Snowmass Letter of Interest: Perturbative Calculations of Anomalous Dimensions in Conformal Field Theories*, 2020.

[20] O. Witzel, *Review on Composite Higgs Models*, *PoS* **LATTICE2018** (2019) 006, [1901.08216].

[21] USQCD collaboration, R. C. Brower, A. Hasenfratz, E. T. Neil, S. Catterall, G. Fleming, J. Giedt et al., *Lattice Gauge Theory for Physics Beyond the Standard Model*, *Eur. Phys. J.* **A55** (2019) 198, [1904.09964].

[22] V. Drach, *Composite electroweak sectors on the lattice*, 2005.01002.

[23] G. Ferretti and D. Karateev, *Fermionic UV completions of Composite Higgs models*, *JHEP* **03** (2014) 077, [1312.5330].

[24] L. Vecchi, *A dangerous irrelevant UV-completion of the composite Higgs*, *JHEP* **02** (2017) 094, [1506.00623].

[25] G. Cacciapaglia, C. Pica and F. Sannino, *Fundamental Composite Dynamics: A Review*, 2002.04914.

[26] Lattice Strong Dynamics collaboration, T. Appelquist, R. Brower, G. Fleming, J. Kiskis, M. Lin, E. Neil et al., *Lattice simulations with eight flavors of domain wall fermions in SU(3) gauge theory*, *Phys. Rev.* **D90** (2014) 114502, [1405.4752].

[27] Lattice Strong Dynamics collaboration, T. Appelquist, R. Brower, G. Fleming, A. Gasbarro, A. Hasenfratz, X.-Y. Jin et al., *Nonperturbative investigations of SU(3) gauge theory with eight dynamical flavors*, *Phys. Rev.* **D99** (2019) 014509, [1807.08411].

[28] Lattice Strong Dynamics collaboration, T. Appelquist, R. C. Brower, K. K. Cushman, G. T. Fleming, A. D. Gasbarro, A. Hasenfratz et al., *Near-conformal dynamics in a chirally broken system*, 2007.01810.

[29] Lattice Strong Dynamics collaboration, T. Appelquist, R. C. Brower, K. K. Cushman, G. T. Fleming, A. D. Gasbarro, A. Hasenfratz et al., *Snowmass Letter of Interest: Composite Higgs from Strong Dynamics on the Lattice*, 2020.

[30] D. B. Kaplan, *Flavor at SSC energies: A new mechanism for dynamically generated fermion masses*, *Nucl. Phys. B* **365** (1991) 259–278.

[31] A. Patella, *A precise determination of the psibar-psi anomalous dimension in conformal gauge theories*, *Phys. Rev.* **D86** (2012) 025006, [1204.4432].

[32] T. DeGrand, Y. Shamir and B. Svetitsky, *Mass anomalous dimension in sextet QCD*, *Phys.Rev.* **D87** (2013) 074507, [1201.0935].

[33] E. Itou and A. Tomiya, *Determination of the mass anomalous dimension for $N_f = 12$ and $N_f = 9$ $SU(3)$ gauge theories*, *PoS* **LATTICE2014** (2014) 252, [1411.1155].

[34] G. Bergner, P. Giudice, G. Münster, I. Montvay and S. Piemonte, *Spectrum and mass anomalous dimension of SU(2) adjoint QCD with two Dirac flavors*, *Phys. Rev.* **D96** (2017) 034504, [1610.01576].

[35] A. Carosso, A. Hasenfratz and E. T. Neil, *Nonperturbative Renormalization of Operators in Near-Conformal Systems Using Gradient Flows*, *Phys. Rev. Lett.* **121** (2018) 201601, [1806.01385].

[36] A. Hasenfratz and O. Witzel, *The running anomalous dimension of fermion bi- and trilinear operators in the 10-flavor SU(3) system*, 2020, in preparation.

[37] O. Antipin and F. Sannino, *Conformal Window 2.0: The large N_f safe story*, *Phys. Rev.* **D97** (2018) 116007, [1709.02354].

[38] P. de Forcrand, S. Kim and W. Unger, *Conformality in many-flavour lattice QCD at strong coupling*, *JHEP* **02** (2013) 051, [1208.2148].

[39] V. Leino, T. Rindlisbacher, K. Rummukainen, F. Sannino and K. Tuominen, *Safety versus triviality on the lattice*, 1908.04605.

[40] A. Nada and A. Ramos, *An analysis of systematic effects in finite size scaling studies using the gradient flow*, 2007.12862.

[41] T.-W. Chiu, *The β-function of $SU(3)$ gauge theory with $N_f = 10$ massless fermions in the fundamental representation*, 1603.08854.

[42] A. Deuzeman, M. P. Lombardo and E. Pallante, *The Physics of eight flavours*, *Phys. Lett.* **B670** (2008) 41–48, [0804.2905].

[43] A. Hasenfratz, D. Schaich and A. Veernala, *Nonperturbative β function of eight-flavor SU(3) gauge theory*, *JHEP* **06** (2015) 143, [1410.5886].

[44] Z. Fodor, K. Holland, J. Kuti, S. Mondal, D. Nogradi and C. H. Wong, *The running coupling of 8 flavors and 3 colors*, *JHEP* **06** (2015) 019, [1503.01132].

[45] A. Hasenfratz and O. Witzel, *Continuous renormalization group β function from lattice simulations*, *Phys. Rev.* **D101** (2020) 034514, [1910.06408].

[46] A. Hasenfratz and O. Witzel, *Dislocations under gradient flow and their effect on the renormalized coupling*, 2004.00758.

[47] P. A. Baikov, K. G. Chetyrkin and J. H. Kühn, *Five-Loop Running of the QCD coupling constant*, *Phys. Rev. Lett.* **118** (2017) 082002, [1606.08659].

[48] T. A. Ryttov and R. Shrock, *Infrared Zero of β and Value of γ_m for an SU(3) Gauge Theory at the Five-Loop Level*, *Phys. Rev.* **D94** (2016) 105015, [1607.06866].

[49] T. A. Ryttov and R. Shrock, *Higher-order scheme-independent series expansions of $\gamma_{\bar{\psi}\psi,IR}$ and β'_{IR} in*

conformal field theories, *Phys. Rev.* **D95** (2017) 105004, [`1703.08558`].

[50] D. Poland, S. Rychkov and A. Vichi, *The Conformal Bootstrap: Theory, Numerical Techniques, and Applications*, *Rev. Mod. Phys.* **91** (2019) 015002, [`1805.04405`].

[51] D. F. Litim, *Fixed points of quantum gravity*, *Phys. Rev. Lett.* **92** (2004) 201301, [`hep-th/0312114`].

[52] J. Rantaharju, C. Pica and F. Sannino, *Ideal Walking Dynamics via a Gauged NJL Model*, *Phys. Rev.* **D96** (2017) 014512, [`1704.03977`].

[53] J. Rantaharju, C. Pica and F. Sannino, *Walking Dynamics Guaranteed*, `1910.02988`.

[54] S. Catterall, D. Schaich, P. H. Damgaard, T. DeGrand and J. Giedt, *N=4 Supersymmetry on a Space-Time Lattice*, *Phys. Rev. D* **90** (2014) 065013, [`1405.0644`].

[55] S. Catterall and D. Schaich, *Lifting flat directions in lattice supersymmetry*, *JHEP* **07** (2015) 057, [`1505.03135`].

[56] D. Schaich, *Progress and prospects of lattice supersymmetry*, *PoS* **LATTICE2018** (2019) 005, [`1810.09282`].

[57] A. Kovner and B. Svetitsky, *Interaction potential in compact three-dimensional QED with mixed action*, *Phys. Rev. D* **60** (1999) 105032, [`hep-lat/9811015`].

[58] B. Svetitsky, O. Raviv and Y. Shamir, *Beta function of three-dimensional QED*, *PoS* **LATTICE2014** (2014) 051, [`1410.0118`].

[59] N. Karthik and R. Narayanan, *Scale-invariance of parity-invariant three-dimensional QED*, *Phys. Rev. D* **94** (2016) 065026, [`1606.04109`].

[60] A. Carosso, A. Hasenfratz, E. Neil, B. Svetitsky and O. Witzel, *The continuous beta function of three-dimensional QED with two and eight flavors*, 2020.

[61] A. Carosso, *Stochastic Renormalization Group and Gradient Flow*, *JHEP* **01** (2020) 172, [`1904.13057`].

[62] A. Carosso, A. Hasenfratz and E. T. Neil, *Stochastic Renormalization Group and Gradient Flow in Scalar Field Theory*, in *37th International Symposium on Lattice Field Theory*, 12, 2019. `1912.01766`.