High-energy scattering and the problem of quantum gravity

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Abstract

Analyzing properties of scattering amplitudes provides a powerful window into a theory, and recent advances have revealed intriguing new structure for the amplitudes of gravity. While nonrenormalizability is a longstanding question, investigating the high-energy scattering behavior of gravity reveals an apparently more profound problem of unitarity, connected to long-distance behavior. It can be seen that *sums* of diagrams become important in this regime. Ultimately one encounters the sum of diagrams that builds up a black hole geometry, and a perturbative analysis about this geometry violates unitarity. A key question is what new physics unitarizes these amplitudes. Study of analytic properties reveals hints through apparent nonpolynomial behavior connected with nonlocality. If new mathematical structure, such as that of the double copy, is part of the fundamental structure of gravity, it is important to understand what it tells us in this regime.

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The problem of reconciling gravity with quantum principles remains one of the most difficult problems of physics. Studying properties of scattering and the S-matrix in a theory can provide a powerful method to reveal its behavior. In particular, recent developments in the study of amplitudes suggest a deeper investigation of gravitational scattering, and its implications for the fundamental properties of quantum gravity.

At the perturbative level, there have been powerful new results in the study of gravity. These include those of the generalized unitarity methods and the beautiful double copy structure (for a review see [1]) where perturbative-level integrands of amplitudes are double copies of the kinematic structure already present in the amplitudes of Yang-Mills. Moreover, these and related methods have provided a powerful way to study the possible structure of perturbative infinities, and the renormalizability structure of the theory (for recent developments and further references see [2]).

While nonrenormalizability has historically been perceived as the central problem of quantum gravity, there has been a growing realization that the problem of unitarity is likely an even more profound one. In particular, there are strong indications it involves not only the short-distance (*e.g.* Planck length) structure of the theory, but also that at longer and in principle macroscopic distances.

A complete description of scattering in a theory includes that of high-energy scattering, and the preceding realization can be understood by considering the high-energy, ultraplanckian, limit of scattering. There, in a semiclassical description, one expects to make a black hole. A perturbative treatment of its evolution [3] then implies an apparently unphysical breakdown of unitarity, revealing an apparent crisis in physics.

To investigate this, and the connection with other scattering behavior, more closely, consider high energy scattering as a function of impact parameter; this is also closely related to a description in terms of momentum transfer q. It is also illuminating to consider this in a general spacetime dimension D, with Newton constant G_D. (For a more extensive discussion, see [4]; earlier work in the context of string theory includes [5–7]). Even at ultraplanckian center-of-mass energy E, gravitational scattering is very weak and dominated by tree or Born level exchange for sufficiently large impact parameter b, specifically as long as $\chi \sim G_D E^2/b^{D-4} \ll 1$. However, the Born approximation fails at the boundary of this regime. For fixed ultraplanckian energy and smaller impact parameters, the sum of ladder diagrams, corresponding to iterated single-graviton exchange, unitarizes the amplitudes; these of course sum to give the eikonal amplitudes [8], [5–7, 9–11].

This regime corresponds to classical scattering, analogous to that of macroscopic objects like planets. It moreover begins to illuminate some important new features. Specifically, the scattering is dominated by a saddlepoint, with total transverse momentum transfer given by $q_\perp \sim \partial\chi/\partial b$. While this momentum transfer can also be superplanckian, an important feature of gravity is that of *momentum fractionation* [12]: the total momentum transfer is carried by the large number of graviton exchanges [13], such that each exchanged graviton carries a typical momentum $k \sim 1/b$ that is subplanckian.

This suggests some important conclusions [12]. First, even for very high ultraplanckian momentum transfers, it is not the properties of individual diagrams that governs the scattering behavior; rather it is the sum of diagrams that yields the dominant saddlepoint. Secondly, the momentum transfer $k \sim 1/b$ of the individual exchanged gravitons indicates that the process is only probing large distances $\sim b$ – not subplanckian distances, despite the superplanckian E and q. This is not strongly dependent on the short-distance structure of individual diagrams. These two observations combined strongly suggest that the issue of renormalizability – which is one of this short distance behavior of diagrams – does not play a central role in this ultraplanckian regime. This high energy behavior emphasizes the need to understand properties of *sums* of diagrams, not just individual diagrams.

While radiation and other effects can also begin to make important contributions at decreasing impact parameter, particularly important effects occur in the vicinity of the impact parameter $b \sim R(E) \sim (G_D E)^{1/(D-3)}$. In this regime, classical scattering leads to black hole formation [14], and one can argue that this classical process provides a starting point for a semiclassical treatment of the scattering. Indeed, at the diagrammatic level, this regime is associated with a sum of a new class of diagrams becoming important, those consisting of graviton tree diagrams inserted between the high-energy legs; in fact, Duff [15] has shown that summing such diagrams builds up a black hole geometry.

This, however, introduces a new set of problems. First, the perturbative description of the geometry fails at the horizon; the perturbation series appears to diverge. One can nonetheless attempt to treat the scattering in a semiclassical expansion about the classical geometry – similar to one about the eikonal saddle. But, quantization of fluctuations about this geometry leads to failure of unitarity [3]. This is the problem (or crisis) of unitarity in high-energy gravitational scattering, also called the black hole information paradox.

It is expected that restoration of unitarity and resolution of the crisis involves new intrinsically quantum-gravitational effects; thus, a key problem is whether clues inferred from studying the S-matrix for gravity can help shed light on these effects. There are different avenues to pursue.

First is the question of analytic and other properties of the S-matrix. It is worth recalling that simply guessing the correct analytic structure of an S-matrix, the Veneziano amplitude, led to the discovery of string theory. In particular, associated with the long-distance behavior of ultra high energy scattering, one sees indications of novel analytic behavior, such as nonpolynomiality associated with the nonlocality of this behavior [13], [4]. It is important to further study the analytic properties of gravitational scattering, in order to seek further clues.

Another important question is that of whether new aspects of gravitational amplitudes that are found perturbatively, such as the double copy structure, are a part of a deeper structure that also governs the behavior of perturbative sums, or nonperturbative, amplitudes. Hints that this could be the case are found in discoveries of double copy structure for classical solutions, such as that for the Schwarzschild black hole [16]; for further examples and discussion see [1]

In short, while new perturbative methods have provided a powerful way to see a beautiful new structure in perturbative amplitudes, the deeply profound problem of quantum gravity requires going beyond to sums of diagrams and nonperturbative amplitudes. If clues inferred from perturbative amplitudes, or from related studies of the high-energy regime, can be found, that could be very important in solving the problem of quantum gravity.

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