Color-Kinematics Duality, Double-Copy Construction, and the Web of Theories: Letter of Interest for Snowmass 2021

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Scattering amplitudes provide an invariant window into the physical dynamic content of relativistic quantum field theory. Yet traditional methods of computing amplitudes in empirically relevant theories scale with a factorial complexity in external particles and precision. If we aspire to collapse the theoretical uncertainty obscuring new physics hidden at all scales—from the microscopic region probed at high energy colliders, to the effective field theory governing the evolution of large scale cosmological structure—this complexity challenge necessitates new ideas and methods in calculation. Novel approaches like the color-kinematics duality and the associated double-copy construction have drastically simplified the situation, relating both gauge and gravity theory predictions to a much smaller kernel of invariant kinematic data.

These approaches realize an amazing truth: gravitational quantum scattering predictions—the invariant quantum evolution of what distance means in space and time, consistent in the classical limit with Einstein's General Relativity—are much simpler than expected. This simplicity can be traced to the fact that their perturbative dynamics are completely encoded in the amplitudes of much simpler gluonic or gauge theories. Surprisingly, these gauge theory predictions are constrained by a structure entirely hidden in any standard ways of writing their actions.

We have known since the 1980s, from string theory limits, that color-stripped gauge theory semi-classical (tree-level) amplitudes encode gravitational amplitudes [1, 2]. This is often informally referred to as the idea that "Gravity is the square of Yang-Mills." Ref. [3] identified an off-shell story relating these predictions, one that generalizes to quantum (multi-loop) corrections [4].

The key idea is that for gauge theory predictions (by definition containing both color and kinematic components), one can find representations where term by term kinematic weights obey the same algebraic relations as generic color weights. Such kinematic weights are said to be color-dual. This makes such dynamics and charges interchangeable and indeed realizes graviton (spin-2) scattering, in asymptotically flat space, as gluonic (spin-1) predictions whose *charges* are—operationally—the kinematics of gluons (spin-1). For a pedagogic set of lecture notes on amplitude calculation using double-copy see [5], as well as a very recent review of double-copy construction, associated relations between novel theories, and applications [6].

This discovery has already trivialized previously intractable analysis. One recent example is the explicit calculation of the high-energy (UV) behavior of maximally supersymmetric supergravity at the five-loop order[7, 8]. This analysis allowed, in turn, the discovery of a new recursive structure, solely observable in the UV, suggesting that even eight-loops may be within reach.

Beyond calculation, this idea of dual strands of weights—woven locally—exposes a previously unrecognized unifying structure in a wide swath of QFT predictions. While first discovered in supersymmetric theories, and indeed for technical reasons they often remain the easiest to calculate in, it is worth emphasizing: this structure is *independent of the presence of supersymmetry* — relevant to phenomenological models that have no superpartners. We now know

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that the scattering amplitudes of many theories can disentangled into two weights along such color-dual algebraic lines. Seemingly disparate theories turn out to be deeply connected by possessing shared weights. For example, Volkov-Akulov fermions and the Special Galilean predictions are related by both theories sharing a kinematic weight associated with non-linear sigma model pions. The entire set of theories manifesting such double-copy structure, referred to as the double-copy web of theories, range from the completely formal—both tree-level open strings and closed strings admit [9–13] field theory double-copy descriptions—to our most phenomenological field theories like QCD with massive fermions in the fundamental [14], and spontaneously broken chiral pions [10, 15]. Unlike holography, this is a weak-weak relationship, order by order in the coupling, one agnostic to dimension.

This structure means relativistic quantum field theory predictions are far more constrained than previously expected with drastic simplifications at both tree and loop level. This has directly led to many new understandings including the scattering equations [16] and related twistorial connected formalisms [17], manifest exposure at the level of solutions to the equations of motion and the action [18–21], as well as discoveries of novel compositional structure in the higher-derivative corrections to effective field theories [22, 23]. It has also begun to offer new interpretations of classical solutions to Einstein's equations (see e.g. [24–48]), a point that is relevant to both gravitational wave physics and cosmology.

Can all quantum questions about the dynamics of space and time be answered in gauge theories? This is an open and tantalizing question at the heart of double-copy studies. Key open questions reach from scattering amplitudes to actively engage quantum field theory communities in formal mathematics, string theory, model building, collider phenomenology, astrophysics, and cosmology. It is difficult to over-emphasize the possibilities opened by realizing a hidden structure relating many quantum field theories—one that is so conceptually suggestive and that so clearly advances the predictive reach of explicit calculation.

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[1] H. Kawai, D.C. Lewellen, and S.H.H. Tye, "A Relation Between Tree Amplitudes of Closed and Open Strings," Nucl. Phys. B **269**, 1–23 (1986).

[2] Frits A. Berends, W.T. Giele, and H. Kuijf, "On relations between multi - gluon and multigraviton scattering," Phys. Lett. B **211**, 91–94 (1988).

[3] Z. Bern, J. J. M. Carrasco, and Henrik Johansson, "New Relations for Gauge-Theory Amplitudes," Phys. Rev. **D78**, 085011 (2008), arXiv:0805.3993 [hep-ph].

[4] Zvi Bern, John Joseph M. Carrasco, and Henrik Johansson, "Perturbative Quantum Gravity as a Double Copy of Gauge Theory," Phys. Rev. Lett. **105**, 061602 (2010), arXiv:1004.0476 [hep-th].

[5] John Joseph M. Carrasco, "Gauge and Gravity Amplitude Relations," in *Proceedings, Theoretical Advanced Study Institute in Elementary Particle Physics: Journeys Through the Precision Frontier: Amplitudes for Colliders (TASI 2014): Boulder, Colorado, June 2-27, 2014*, WSP (WSP, 2015) pp. 477–557, arXiv:1506.00974 [hep-th].

[6] Zvi Bern, John Joseph Carrasco, Marco Chiodaroli, Henrik Johansson, and Radu Roiban, "The Duality Between Color and Kinematics and its Applications," (2019), arXiv:1909.01358 [hep-th].

[7] Zvi Bern, John Joseph M. Carrasco, Wei-Ming Chen, Henrik Johansson, Radu Roiban, and Mao Zeng, "Five-loop four-point integrand of $N = 8$ supergravity as a generalized double copy," Phys. Rev. **D96**, 126012 (2017), arXiv:1708.06807 [hep-th].

[8] Zvi Bern, John Joseph Carrasco, Wei-Ming Chen, Alex Edison, Henrik Johansson, Julio Parra-Martinez, Radu Roiban, and Mao Zeng, "Ultraviolet Properties of $\mathcal{N} = 8$ Supergravity at Five Loops," Phys. Rev. **D98**, 086021 (2018), arXiv:1804.09311 [hep-th].

[9] Johannes Broedel, Oliver Schlotterer, Stephan Stieberger, and Tomohide Terasoma, "All order α'-expansion of superstring trees from the Drinfeld associator," Phys. Rev. D **89**, 066014 (2014), arXiv:1304.7304 [hep-th].

[10] John Joseph M. Carrasco, Carlos R. Mafra, and Oliver Schlotterer, "Abelian Z-theory: NLSM amplitudes and α'-corrections from the open string," JHEP **06**, 093 (2017), arXiv:1608.02569 [hep-th].

[11] Carlos R. Mafra and Oliver Schlotterer, "Non-abelian Z-theory: Berends-Giele recursion for the α'-expansion of disk integrals," JHEP **01**, 031 (2017), arXiv:1609.07078 [hep-th].

[12] John Joseph M. Carrasco, Carlos R. Mafra, and Oliver Schlotterer, "Semi-abelian Z-theory: NLSM+ϕ^3 from the open string," JHEP **08**, 135 (2017), arXiv:1612.06446 [hep-th].

[13] Thales Azevedo, Marco Chiodaroli, Henrik Johansson, and Oliver Schlotterer, "Heterotic and bosonic string amplitudes via field theory," JHEP **10**, 012 (2018), arXiv:1803.05452 [hep-th].

[14] Henrik Johansson and Alexander Ochirov, "Color-Kinematics Duality for QCD Amplitudes," JHEP **01**, 170 (2016), arXiv:1507.00332 [hep-ph].

[15] Freddy Cachazo, Song He, and Ellis Ye Yuan, "Scattering Equations and Matrices: From Einstein To Yang-Mills, DBI and NLSM," JHEP **07**, 149 (2015), arXiv:1412.3479 [hep-th].

[16] Freddy Cachazo, Song He, and Ellis Ye Yuan, "Scattering equations and Kawai-Lewellen-Tye orthogonality," Phys. Rev. **D90**, 065001 (2014), arXiv:1306.6575 [hep-th].

[17] Lionel Mason and David Skinner, "Ambitwistor strings and the scattering equations," JHEP **07**, 048 (2014),

arXiv:1311.2564 [hep-th].
[18] Zvi Bern, Tristan Dennen, Yu-tin Huang, and Michael Kiermaier, "Gravity as the Square of Gauge Theory," Phys. Rev. D **82**, 065003 (2010), arXiv:1004.0693 [hep-th].
[19] Mathias Tolotti and Stefan Weinzierl, "Construction of an effective Yang-Mills Lagrangian with manifest BCJ duality," JHEP **07**, 111 (2013), arXiv:1306.2975 [hep-th].
[20] Elliot Bridges and Carlos R. Mafra, "Algorithmic construction of SYM multiparticle superfields in the BCJ gauge," JHEP **10**, 022 (2019), arXiv:1906.12252 [hep-th].
[21] Leron Borsten, Branislav Jurˇ co, Hyungrok Kim, Tommaso Macrelli, Christian Saemann, and Martin Wolf, "BRST-Lagrangian Double Copy of Yang-Mills Theory," (2020), arXiv:2007.13803 [hep-th].
[22] John Joseph M. Carrasco, Laurentiu Rodina, Zanpeng Yin, and Suna Zekioglu, "Simple encoding of higher derivative gauge and gravity counterterms," (2019), arXiv:1910.12850 [hep-th].
[23] Ian Low and Zhewei Yin, "New Flavor-Kinematics Dualities and Extensions of Nonlinear Sigma Models," Phys. Lett. B **807**, 135544 (2020), arXiv:1911.08490 [hep-th].
[24] Ricardo Monteiro, Donal O'Connell, and Chris D. White, "Black holes and the double copy," JHEP **12**, 056 (2014), arXiv:1410.0239 [hep-th].
[25] Andrs Luna, Ricardo Monteiro, Donal O'Connell, and Chris D. White, "The classical double copy for TaubNUT spacetime," Phys. Lett. **B750**, 272–277 (2015), arXiv:1507.01869 [hep-th].
[26] Ricardo Monteiro, Donal O'Connell, and Chris D. White, "Gravity as a double copy of gauge theory: from amplitudes to black holes," *Proceedings, 7th Black Holes Workshop 2014: Aveiro, Portugal, December 18-19, 2014*, Int. J. Mod. Phys. **D24**, 1542008 (2015).
[27] Andrs Luna, Ricardo Monteiro, Isobel Nicholson, Donal O'Connell, and Chris D. White, "The double copy: Bremsstrahlung and accelerating black holes," JHEP **06**, 023 (2016), arXiv:1603.05737 [hep-th].
[28] Walter D. Goldberger, Siddharth G. Prabhu, and Jedidiah O. Thompson, "Classical gluon and graviton radiation from the bi-adjoint scalar double copy," Phys. Rev. **D96**, 065009 (2017), arXiv:1705.09263 [hep-th].
[29] Walter D. Goldberger and Alexander K. Ridgway, "Radiation and the classical double copy for color charges," Phys. Rev. **D95**, 125010 (2017), arXiv:1611.03493 [hep-th].
[30] Andrs Luna, Ricardo Monteiro, Isobel Nicholson, Alexander Ochirov, Donal O'Connell, Niclas Westerberg, and Chris D. White, "Perturbative spacetimes from Yang-Mills theory," JHEP **04**, 069 (2017), arXiv:1611.07508 [hep-th].
[31] Mariana Carrillo-Gonzlez, Riccardo Penco, and Mark Trodden, "The classical double copy in maximally symmetric spacetimes," JHEP **04**, 028 (2018), arXiv:1711.01296 [hep-th].
[32] Mariana Carrillo Gonzlez, Riccardo Penco, and Mark Trodden, "Radiation of scalar modes and the classical double copy," JHEP **11**, 065 (2018), arXiv:1809.04611 [hep-th].
[33] Walter D. Goldberger and Alexander K. Ridgway, "Bound states and the classical double copy," Phys. Rev. **D97**, 085019 (2018), arXiv:1711.09493 [hep-th].
[34] Walter D. Goldberger, Jingping Li, and Siddharth G. Prabhu, "Spinning particles, axion radiation, and the classical double copy," Phys. Rev. **D97**, 105018 (2018), arXiv:1712.09250 [hep-th].
[35] Andrs Luna, Isobel Nicholson, Donal O'Connell, and Chris D. White, "Inelastic Black Hole Scattering from Charged Scalar Amplitudes," JHEP **03**, 044 (2018), arXiv:1711.03901 [hep-th].
[36] Ricardo Monteiro, Isobel Nicholson, and Donal O'Connell, "Spinor-helicity and the algebraic classification of higher-dimensional spacetimes," Class. Quant. Grav. **36**, 065006 (2019), arXiv:1809.03906 [gr-qc].
[37] Andrés Luna, Ricardo Monteiro, Isobel Nicholson, and Donal O'Connell, "Type D Spacetimes and the Weyl Double Copy," Class. Quant. Grav. **36**, 065003 (2019), arXiv:1810.08183 [hep-th].
[38] N. E. J. Bjerrum-Bohr, Poul H. Damgaard, Guido Festuccia, Ludovic Plant, and Pierre Vanhove, "General Relativity from Scattering Amplitudes," Phys. Rev. Lett. **121**, 171601 (2018), arXiv:1806.04920 [hep-th].
[39] Clifford Cheung, Ira Z. Rothstein, and Mikhail P. Solon, "From Scattering Amplitudes to Classical Potentials in the Post-Minkowskian Expansion," Phys. Rev. Lett. **121**, 251101 (2018), arXiv:1808.02489 [hep-th].
[40] David A. Kosower, Ben Maybee, and Donal O'Connell, "Amplitudes, Observables, and Classical Scattering," JHEP **02**, 137 (2019), arXiv:1811.10950 [hep-th].
[41] Ben Maybee, Donal O'Connell, and Justin Vines, "Observables and amplitudes for spinning particles and black holes," JHEP **12**, 156 (2019), arXiv:1906.09260 [hep-th].
[42] Nima Arkani-Hamed, Yu-tin Huang, and Donal O'Connell, "Kerr black holes as elementary particles," JHEP **01**, 046 (2020), arXiv:1906.10100 [hep-th].
[43] Zvi Bern, Clifford Cheung, Radu Roiban, Chia-Hsien Shen, Mikhail P. Solon, and Mao Zeng, "Scattering Amplitudes and the Conservative Hamiltonian for Binary Systems at Third Post-Minkowskian Order," Phys. Rev. Lett. **122**, 201603 (2019), arXiv:1901.04424 [hep-th].
[44] Zvi Bern, Clifford Cheung, Radu Roiban, Chia-Hsien Shen, Mikhail P. Solon, and Mao Zeng, "Black Hole Binary Dynamics from the Double Copy and Effective Theory," JHEP **10**, 206 (2019), arXiv:1908.01493 [hep-th].
[45] Mariana Carrillo González, Riccardo Penco, and Mark Trodden, "Shift symmetries, soft limits, and the double copy beyond leading order," (2019), arXiv:1908.07531 [hep-th].
[46] Yu-Tin Huang, Uri Kol, and Donal O'Connell, "Double copy of electric-magnetic duality," Phys. Rev. D **102**, 046005 (2020), arXiv:1911.06318 [hep-th].
[47] Zvi Bern, Harald Ita, Julio Parra-Martinez, and Michael S. Ruf, "Universality in the classical limit of massless gravitational scattering," Phys. Rev. Lett. **125**, 031601 (2020), arXiv:2002.02459 [hep-th].
[48] Zvi Bern, Andres Luna, Radu Roiban, Chia-Hsien Shen, and Mao Zeng, "Spinning Black Hole Binary Dynamics, Scattering

Amplitudes and Effective Field Theory," (2020), arXiv:2005.03071 [hep-th].