Snowmass2021 - Letter of Interest

On-Shell Methods for the SMEFT

Thematic Areas:

(TF02) Effective field theory techniques
(TF04) Scattering amplitudes
(TF06) Theory techniques for precision physics
(TF07) Collider phenomenology

Contact Information:

Enrico Herrmann: eh10@stanford.edu
Julio Parra-Martinez: jparra@physics.ucla.edu

Authors: Rafael Aoude,[1] Brad Bachu,[2] Andreas Helset,[3] Enrico Herrmann,[4] Tzu-Chen Huang,[3] Elizabeth Jenkins,[5] Seth Koren,[6,7] Da Liu,[8] Aneesh Manohar,[5] Julio Parra-Martinez,[3,9] Grant N. Remmen,[6,10] Nicholas L. Rodd,[11,12] Eric Sawyer,[9] Chia-Hsien Shen,[9] Akshay Yelleshpur,[2] Zhewei Yin.[13]

Abstract: Letter of interest for the 2021 Snowmass planning process within the "scattering amplitudes" section of the Theory Frontier (TF10). We are expressing our excitement about various formal and phenomenological prospects of on-shell methods for the Standard Model as an effective field theory.

[1]PRISMA+ Cluster of Excellence & Mainz Institute of Theoretical Physics, Johannes Gutenberg-Universität Mainz, 55099 Mainz, Germany
[2]Department of Physics, Princeton University, Princeton, NJ 08540, USA
[3]Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena, CA 91125
[4]SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94039, USA
[5]Department of Physics, University of California, San Diego, CA 92093-0319, USA
[6]Department of Physics, University of California, Santa Barbara, CA 93106, USA
[7]Enrico Fermi Institute, University of Chicago, Chicago, IL 60637, USA
[8]Center for Quantum Mathematics and Physics (QMAP), University of California, Davis, CA 95616, USA
[9]Mani L. Bhaumik Institute for Theoretical Physics, University of California, Los Angeles, CA 90095, USA
[10]Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106, USA
[11]Berkeley Center for Theoretical Physics, University of California, Berkeley, CA 94720, USA
[12]Theoretical Physics Group, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA
[13]Department of Physics and Astronomy, Northwestern University, Evanston, IL 60208, USA

The Standard Model, arguably one of the milestone achievements of twentieth century physics, has withstood numerous experimental tests thus far. With the discovery of the Higgs boson [1, 2], the complete particle content of the theory has been confirmed experimentally and tested up to energy scales beyond electroweak (EW) symmetry breaking. The absence of direct experimental evidence for new particles at the large hadron collider (LHC), and other experimental facilities, strengthens the viability of the Standard Model as a description of fundamental physics at even higher energies. Despite this phenomenal success story, there are various reasons (dark matter, dark energy, and the matter-antimatter asymmetry in our Universe, to name a few) to believe that the Standard Model is not the end of the road. As such, it is reasonable to view the Standard Model as an effective field theory (EFT) approximation of our world valid up to the TeV scale. From this perspective, we can parameterize our ignorance about the physics at energy scales beyond the current reach, in a model-independent way, by systematically writing down possible deviations from the Standard Model in terms of higher-dimensional operators and their respective Wilson coefficients [3–6], which can be constrained experimentally. The theory including the higher-dimension operators is known as the Standard Model EFT (SMEFT) [7]. Besides the important phenomenological questions about the correct fundamental description of our Universe, in recent decades we have seen enormous progress in our understanding of quantum field theories (QFTs). Within any QFT, *scattering amplitudes* are central objects of interest, and a growing field of researchers set out to study their deeper structures. This has led to numerous advances such as efficient recursive methods [8, 9], new hidden symmetries [10–13], unexpected connections between different and seemingly unrelated theories [14–17], all the way to novel geometric ideas underlying perturbative QFT [18, 19]. We collectively summarize these advances under the term *on-shell methods*. Most of these ideas were first explored for scattering events involving massless particles in minimally coupled theories. In the real world of the Standard Model, however, some particles are massive. Fortunately, on-shell methods are now ripe to include the effects of masses and higher-dimension operators. In fact a program has already been started in this direction, by cataloguing on-shell low-point amplitudes [20–23], which avoid gauge and field redefinition ambiguities inherent to traditional Lagrangian approaches.

In this LOI, we therefore argue for further explorations into both phenomenological and formal studies of the structure of scattering amplitudes and the SMEFT taking masses into account. In the following, we are going to list and briefly describe various interesting avenues of research within the coming years. We hope these will serve as starting point for further discussion within the Snowmass process.

Constraining the space of consistent EFTs —The motivation underlying the SMEFT is to use known physics as a scaffold for the new, packaging high energy (UV) effects as Wilson coefficients at low energies (in the IR). Building on the S-matrix program, it has been shown [24] that EFT coefficients cannot be chosen arbitrarily without violating basic QFT principles: causality, locality, and unitarity. Recently, this has been applied to the SMEFT itself [25–28], placing powerful model-independent constraints on the possible deviations from the Standard Model that could appear at collider or precision measurements. Operators mediating (e.g., CP or flavor) violation processes are provably bounded by closely related operators that conserve these properties, with consequences for—and unprecedented connections between—experiments searching for the two types of effects. Much work remains. Model-independent results so far are largely restricted to subsets of the dimension-8 SMEFT. Open questions include extending these results to other operators (e.g., B or L violation), bounding operators at dimension-6, and probing the link between causality and unitarity throughout this story. The ultimate goal of this program would be to map the full boundary of the SMEFT, which would place an important theoretical prior on upcoming experiments and increase the statistical power of limits on the SMEFT. In the event of a discovery of deviations from the Standard Model, it immediately allows for robust predictions relating different probes of new physics and gives invaluable insight into the nature of the UV theory. In a related effort, novel insights into the structure of massive scattering amplitudes point towards a geometric picture that underlies the effectiveness of modern amplitude techniques. The so-called *EFThedron* constrains the parameter space of theories beyond the optical theorem. It does so by exploiting a new geometric structure behind the Wilson coefficients in the low energy effective field theories whose UV completion satisfies certain assumptions. Such structure has been applied to a variety of problems [29–31] but it remains an important question to understand its implications for the SMEFT.

On-shell insights into structure of the renormalization group—In the EFT framework, matching new physics to the SMEFT Wilson coefficients is carried out at a high-energy scale $\Lambda \gg 1$ TeV, while current measurements are performed at various low-energy scales. Combining constraints from different experiments therefore requires proper renormalization group evolution of operators in the SMEFT to the appropriate scale. Here, on-shell methods can play a key role. Recently, the full one-loop anomalous dimension matrix of dimension-six operators was calculated [32–34], wherein a number of unexpected zeros were discovered [35]. These one-loop zeros were subsequently explained via on-shell methods, using helicity [36] and angular momentum [37] selection rules. More precise applications of the SMEFT require access to the full one-loop SMEFT matrix elements in order to perform matching calculations, and computing two-loop anomalous dimensions for the running of the Wilson coefficients. On-shell methods are particularly adept at contributing to this problem, as unitary cuts can directly access the running by calculating the coefficient of $\log(\mu)$ using renormalized amplitudes as input [38]. The simplicity of on-shell methods may provide more insights into the structure of the anomalous dimension matrix. A hint that this is the case can be found in recent computations, which using both on-shell and traditional methods showed that a wealth of massless one-loop matrix elements [37, 39] and multi-loop anomalous dimensions [40] are zero, or can be eliminated by a judicious choice of renormalization scheme [41]. With the results at one and two loops, we can explore possible hidden structures beyond vanishing coefficients. As there are a large number of operators in the SMEFT, it is crucial to simplify its structure, and on-shell methods can offer guidance into this endeavor.

Massive tree-level recursion and compact Parke-Taylor type formulae for massive SM amplitudes?—One of the key insights in the modern amplitudes program arose from the fact that locality and unitarity of scattering amplitudes allow for a recursive definition of tree-level S-matrix elements purely in terms of a minimal set of simpler gauge-invariant building blocks [8,9]. Historically, these methods were developed for massless gauge theories and led to a direct and simple proof of the famous one-line Parke-Taylor formula [42] for the scattering of arbitrary numbers of gluons. Parke and Taylor's original computation was motivated by phenomenologically relevant collider physics questions, but subsequently sparked the interest of numerous theorists to explain and harness the underlying simplicity of the final result. It is interesting to investigate whether similar structures and methods exist for massive amplitudes within the Standard Model and beyond. Explicit, compact results like the Parke-Taylor formula would not only have important phenomenological applications, but lead to another boost in understanding the hidden structures behind these massive amplitudes. With the advent of modern massive on-shell helicity methods [43], we are in the perfect position to start exploring these questions beyond existing attempts [44–48].

On-shell understanding of the Higgs mechanism and phases of gauge theories—With the aforementioned massive spinor helicity formalism [43], it is possible to efficiently catalogue three-point amplitudes involving both massless and massive particles. Higher-point amplitudes can, in principle, be constructed by gluing together lower-point amplitudes on factorization channels. However, this does not capture possible contact terms that depend on the physics at higher energies. Consistency among amplitudes at different energy scales enables us to derive relations between couplings and masses that ultimately stem from symmetries of the high energy phase [47]. These techniques can also be used to check which phases of a theory are consistent with one another. Likewise, in [43] it was shown that the Higgs mechanism can be understood as IR unification of different massless helicity amplitudes in the UV. This is complementary to the classic results by Cornwall et al. [49], who proved that the only consistent UV theory of interacting massive scalar, spinor and vector fields is equivalent to a spontaneously broken gauge theory. Within the massive on-shell formalism, it would be nice to reproduce these results by constructing all four-particle amplitudes and imposing tree-level unitarity. We expect a new understanding and perspective will emerge from this study.

Massive amplitudes for current and future colliders—There is a direct need for amplitudes involving many massive Standard Model states in the coming decades. High-energy proton-proton colliders will give a fantastic opportunity to study top quark physics, including events with many outgoing tops. At future lepton colliders, vector boson fusion will be the leading production process for many final states of interest, including production of many EW gauge bosons. The number of Feynman diagrams grows factorially with the number of particles, meaning even tree-level cross sections are usually only understood numerically. These amplitudes require delicate cancellation of potential gauge-dependent energy growing terms across many diagrams (c.f. [50]), which can cause problems for numerical integrators at high energies. The advantage of the on-shell massless and massive amplitude formalism is building all the higher-point amplitudes in terms of on-shell lower-point amplitudes recursively without introducing any gauge redundancy. The ultimate goal here is to formulate an efficient method to calculate amplitudes with multiple massive particles by exploring the on-shell massive formalism developed in [43]. Finding analytic formulae for amplitudes with many massive Standard Model states helps to compute both signal and background at future colliders.

Other EFTs: Finally, on-shell methods are also applicable to effective field theories that are descendants of the SMEFT, such as Low-energy Effective Field Theory [51,52] and Heavy Quark Effective Theory [53]. Recently, HQET and related heavy particle effective theories were formulated using massive on-shell spinor-helicity variables [54]. The application of on-shell methods to other modern EFTs for particle physics, e.g., Soft-Collinear EFT (SCET) [55, 56], will likely bring further insight and streamline calculations.

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