From scattering amplitudes to the relativistic two-body problem

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Abstract

Gravitational wave observatories rely on precise theoretical calculations and accurate waveforms to detect binary mergers from the far reaches of the universe and understand their underlying physics. We propose to leverage recent and ongoing breakthroughs in quantum scattering amplitudes to open new windows of precision in gravitational wave physics and to deepen our understanding of gravity in various regimes.

The complete gravitational evolution of a system of two massive bodies is a difficult and unsolved problem in the entire parameter space. Gravitational radiation from such astrophysical systems, spectacularly observed [1] by the LIGO/Virgo collaborations, provided the latest confirmation of Einstein's General Relativity (GR). The increased precision of future generations of gravitational wave observatories will probe a far more diverse population of binaries, demanding theoretical input of matching qualities. Established analytic GR methods, such as the post-Newtonian (PN) expansion [2] and the self-force formalism [3], deriving observables from Einstein's equations, scale slower than the demands. Numerical solutions [4] have difficulties accessing the complete parameter space and are expensive for long-time evolution.

New approaches to the relativistic two-body problem combine recent and ongoing breakthroughs in quantum scattering amplitudes – at whose core is the focus on physical quantities [5, 6] and relations between gauge and gravity theories [7] – and effective field theories [8, 9]. They were recently shown to be very effective, pushing the state-of-the-art in the conservative dynamics of spinless [10], finite-size [11, 12] and spinning [13] bodies. Such fully relativistic results have been desired for some time [14] in the gravitational wave community.

While the initial conditions in a scattering process are different from those in a bound orbit, which are the predominant source of gravitational wave events, the open-orbit and closed-orbit dynamics are governed by the same action. Thus, it may be determined from the former using the vast array of powerful tools developed for particle physics problems and subsequently used for the latter. Aspects of amplitude-based results have been verified in [15, 16], which also highlight the use of amplitudes integration methods in the PN expansion. A preliminary study of the impact of the new results on gravitational waveforms was carried out by members of the LIGO collaboration in [17] and concluded that a more comprehensive study is warranted. Another framework, which does not separate conservative dynamics and radiation reaction, was described in [18].

The manifestly-relativistic formulation of scattering amplitudes implies that the resulting observables have complete dependence on the (relative) velocity of the binary components. While this aids in the quest for precision, its most important consequences lie elsewhere. On the one hand it is essential for the construction of waveforms for "fly-by events", in which the velocity is not bound by the virial theorem. On the other, by manifesting the analytic structure of each order in perturbation theory, they offer new insight into features of gravitational perturbation theory, expose hereto unexpected structure in certain observables, and may offer a path to resummation of perturbation theory in the classical limit, thus opening a window into gravity's strong field regime.

We propose to explore the range and depth of these new methods. They complement GR-based approaches by offering a new generation of theoretical input for the detection and understanding of merger events and deepen our fundamental understanding of gravitational interactions and its consistent extensions.

New perspectives on observables: General coordinate invariance precludes the existence of local observables. Scattering amplitudes suggest a natural class of observables, defined at asymptotic infinity. They can be constructed either from the solution to Hamilton's equations of an effective two-body Hamiltonian or directly in terms of the scattering matrix.

A distinct link between observables and scattering amplitudes comes [18] through the study of the classical limit of relevant quantum observables and bypasses the use of a Hamiltonian.

It is important to understand how to directly relate observables characteristic to bound systems and scattering data and thus to create new methods for bound-state calculations in quantum and classical theory. Preliminary work in this direction, relying on certain analytic continuations was carried out in [19]. More generally, it is important to identify and develop the theoretical tools needed to fully exploit gravitational-wave observations in the quest to understand gravitating compact objects and black hole horizons.

Ever-higher perturbative orders: With the demonstrated power and expected scalability of amplitude-based approaches, it is important to evaluate further orders in the conservative two-body Hamiltonians for both spinless and spinning bodies. Connection to experimental applications requires that the formalism used to account for conservative contributions be extended to include dissipative effects. Application of the framework [18] to current observations in gravitational wave physics requires it be extended to bound systems. The precise gravitational wave templates and renewed understanding of aspects of gravity that will follow from such extensions and from higher-order calculations are clearly worthwhile.

Apart from obvious applications to LIGO/Virgo physics, new higher-order results will bring a post-Minkowskian perspective on interesting aspects which are known to appear from a PN analysis. Their understanding, perhaps inspired by that developed in the PN approximation, will clarify their consequences at even higher orders.

Gedanken higher-order calculations contain a wealth of information even before the actual calculations are completed; for example, the momentum dependence of the final result is governed by the nontrivial loop integrals that can appear. Such partial information can aid the matching with numerical GR calculations which, for the time being, is the only reliable approach to the final moments of a binary inspiral. In turn, such a detailed fit may offer further analytic insight into this period of the binary's evolution.

Higher-order structure and resummation: Exact results in interacting theories, even in the classical regime, are rare. Patterns exposed by explicit higher perturbative orders may hold the key to understanding the structure of perturbation theory and eventually lead to its resummation.

Proof of principle is the conjectured expression relating scattering observables of spinning bodies to the eikonal phase of the corresponding scattering amplitude. The very fact that there exists a single function that potentially captures all classical observables of such processes is surprising. Tests of such conjectures may exploit special configurations of particles, new perturbative expansions which access different regions of parameter space, etc.

Another path to exact results goes through enhancements of perturbative calculations. The framework of the effective one-body theory (EOB) [20] provides a model as, apart from perturbative data, it also incorporates exact results available for systems with extreme mass ratios. In general, it is critical that information in distinct regions of parameter space be available. The amount of physics contained in all-order results is remarkable and includes, among others, possible analytic access to horizon formation in the binary coalescence.

All of these and other open questions in the study of the relativistic two-body problem would benefit very much from detailed analysis using new quantum field theory methods.

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