# Manifestly Causal Scattering Amplitudes

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We are very much interested in the development of the numerical evaluation of multi-loop scattering amplitudes. In particular, our efforts are focalised on the improvement of the loop-tree duality (LTD) theorem. This formalism, that started in 2008 [1, 2], aims at opening loops into connected trees and, hence, perform an evaluation of the phase-space integrals. LTD features several properties at integrand level that devises the infrared (IR) and ultraviolet (UV) structure of the amplitude. Thus, a local UV renormalisation and IR subtraction is achieved before integrating in four space-time dimensions. Originating, the so-called four-dimensional unsubtraction (FDU) scheme [3, 4]. Within FDU, the individual treatment of UV counter-terms and IR subtraction terms is not carried out in $d$ space-time dimensions. This alternative of integrating out a scattering amplitude in purely four space-time dimensions has successfully worked with cross-sections containing, as building blocks, one-loop Feynman integrals [5, 6, 7]. Furthermore, preliminar results that encompass two-loop Feynman integrals have been also presented [8].

In view of the simplicity and advantages of the LTD theorem, there have been recent developments provided by us [9, 10, 11, 12] as well as alternative reformulations studied by other groups [13, 14, 15, 16, 17]. This treatment of multi-loop Feynman integrals shows that numerical evaluations, together with the consideration of IR and UV singularities, can be revisited. Thus, in partner with a proper automation, revealing efficient new strategies to evaluate integrals that traditional approaches are not able to compute.

Let us briefly comment on the causal representation of Feynman integrals. In fact, scattering amplitudes and loop integrals in the Feynman representation exhibit, in general, unphysical or non-causal singularities in the loop momentum space. Their representation in terms of Feynman parameters, Symanzik polynomials or Mellin-Barnes also inherit the unphysical structure of the original loop integral. However, all these unphysical singularities are expected to cancel after integration. Hence, owing to the multi-loop LTD formulation, it has explicitly been shown that Feynman integrals in momentum space can be cast into expressions free of non-causal or unphysical singularities, allowing, then, for a stable numerical evaluation. Explicit expressions and calculations have been presented in [10, 11].

Continuing with the development of LTD, we plan on working in the following directions:

- Within the multi-loop LTD representation, recently formulated, we have open a new direction towards the numerical evaluation of Feynman integrals. In particular, we elucidate a new structure of Feynman integrals that only accounts for physical singularities, allowing for a stable numerical evaluation.
- On top of the direct evaluation of multi-loop integrals, we also are working on their asymptotic expansion within the LTD formalism. This approach is currently being extending at multi-loop level.

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- So far we have calculated IR safe observables up to two loops. We plan on extending the complete FDU formulation to two loops and, therefore, generalise its prescriptions at multi-loop level.
- The knowledge of the IR structure of scattering amplitudes in $\mathcal{N} = 4$ sYM is of great interest and can be used to profit the features of LTD. In fact, we can obtain new properties that aid at evaluation of physical observables.

We consider that the above-mentioned arguments are relevant for the field of scattering amplitudes and need to be further studied.

## References


[1] S. Catani, T. Gleisberg, F. Krauss, G. Rodrigo and J.-C. Winter, *From loops to trees by-passing Feynman's theorem*, *JHEP* **09** (2008) 065, [0804.3170].

[2] I. Bierenbaum, S. Catani, P. Draggiotis and G. Rodrigo, *A Tree-Loop Duality Relation at Two Loops and Beyond*, *JHEP* **10** (2010) 073, [1007.0194].

[3] G. F. R. Sborlini, F. Driencourt-Mangin, R. Hernandez-Pinto and G. Rodrigo, *Four-dimensional unsubtraction from the loop-tree duality*, *JHEP* **08** (2016) 160, [1604.06699].

[4] G. F. R. Sborlini, F. Driencourt-Mangin and G. Rodrigo, *Four-dimensional unsubtraction with massive particles*, *JHEP* **10** (2016) 162, [1608.01584].

[5] F. Driencourt-Mangin, G. Rodrigo and G. F. Sborlini, *Universal dual amplitudes and asymptotic expansions for $gg \to H$ and $H \to \gamma\gamma$ in four dimensions*, *Eur. Phys. J. C* **78** (2018) 231, [1702.07581].

[6] F. Driencourt-Mangin, G. Rodrigo, G. F. Sborlini and W. J. Torres Bobadilla, *On the interplay between the loop-tree duality and helicity amplitudes*, 1911.11125.

[7] J. Plenter and G. Rodrigo, *Asymptotic expansions through the loop-tree duality*, 2005.02119.

[8] F. Driencourt-Mangin, G. Rodrigo, G. F. R. Sborlini and W. J. Torres Bobadilla, *Universal four-dimensional representation of $H \to \gamma\gamma$ at two loops through the Loop-Tree Duality*, *JHEP* **02** (2019) 143, [1901.09853].

[9] J. J. Aguilera-Verdugo, F. Driencourt-Mangin, J. Plenter, S. Ramírez-Uribe, G. Rodrigo, G. F. Sborlini et al., *Causality, unitarity thresholds, anomalous thresholds and infrared singularities from the loop-tree duality at higher orders*, *JHEP* **12** (2019) 163, [1904.08389].

[10] J. J. Aguilera-Verdugo, F. Driencourt-Mangin, R. J. Hernandez Pinto, J. Plenter, S. Ramirez-Uribe, A. E. Renteria Olivo et al., *Open loop amplitudes and causality to all orders and powers from the loop-tree duality*, *Phys. Rev. Lett.* **124** (2020) 211602, [2001.03564].

[11] J. J. Aguilera-Verdugo, R. J. Hernandez-Pinto, G. Rodrigo, G. F. Sborlini and W. J. Torres Bobadilla, *Causal representation of multi-loop amplitudes within the loop-tree duality*, 2006.11217.

[12] S. Ramirez-Uribe, R. J. Hernandez-Pinto, G. Rodrigo, G. F. Sborlini and W. J. Torres Bobadilla, *Universal opening of four-loop scattering amplitudes to trees*, 2006.13818.

[13] E. Tomboulis, *Causality and Unitarity via the Tree-Loop Duality Relation*, *JHEP* **05** (2017) 148, [1701.07052].

[14] Z. Capatti, V. Hirschi, D. Kermanschah and B. Ruijl, *Loop-Tree Duality for Multiloop Numerical Integration*, *Phys. Rev. Lett.* **123** (2019) 151602, [1906.06138].

[15] Z. Capatti, V. Hirschi, D. Kermanschah, A. Pelloni and B. Ruijl, *Numerical Loop-Tree Duality: contour deformation and subtraction*, *JHEP* **04** (2020) 096, [1912.09291].

[16] R. Runkel, Z. Szőr, J. P. Vesga and S. Weinzierl, *Causality and loop-tree duality at higher loops*, *Phys. Rev. Lett.* **122** (2019) 111603, [1902.02135].

[17] R. Runkel, Z. Szőr, J. P. Vesga and S. Weinzierl, *Integrands of loop amplitudes within loop-tree duality*, 1906.02218.