Towards global fits of three-dimensional hadron structure from lattice QCD

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Unravelling the constituent structure of hadrons–hadron tomography–lies at the intersection of the energy and intensity frontiers. A detailed picture of the collinear structure of protons, encapsulated in parton distribution functions (PDFs), continues to play a dual role at the Large Hadron Collider (LHC). On the one hand, precise PDFs are key to reducing theoretical systematic uncertainties in a range of processes at the energy frontier. On the other, the wealth of data generated by the LHC has clarified our understanding of many PDFs over a broad range of Bjorken-x and momentum transfers. New experiments at Jefferson Lab will complement the kinematic coverage of the LHC at large Bjorken-x, while the EIC will provide fresh insight into the small-x regime. Moreover, understanding the three-dimensional structure and mechanical properties of the nucleon, in terms of generalized parton distributions (GPDs), transverse-momentum dependent distributions (TMDs) and, ultimately, Wigner distributions, will lay bare the origin of nucleon mass and spin, and provide a better foundation for understanding the QCD equation of state, drawing together systems as disparate as light nuclei and neutron stars. This program will require coordinated effort, emphasised by the close connections between the lattice, phenomenology, and experimental communities highlighted in the "Hadronic Tomography at the EIC and the Energy Frontier" LOI.

Progress in PDFs Recent developments in first principles' calculations of x-dependent hadron structure have reinvigorated attempts to calculate the x dependence of PDFs and and their three-dimensional counterparts, GPDs, directly from lattice quantum chromodynamics (QCD). The two most frequently used approaches, large momentum effective theory (LaMET) [1, 2] and short-distance factorization (SDF) [3–5], are based on extracting hadron structure from matrix elements of fields at space-like separations, although there are a suite of alternative methods currently under investigation [6–8]. Promising proof-of-principle calculations have demonstrated the possibilities of this approach [9–11], and now the work of cataloguing and quantifying systematic uncertainties remains [12, 13].

In addition to the usual systematic uncertainties associated with lattice QCD calculations–pion mass, discretisation, excited state and finite volume effects–lattice calculations of PDFs suffer from an unavoidable "inverse problem". The challenge is to determine a continuous function (for example, the PDF as a function of x at a fixed scale μ^2) from the Fourier transform of a limited set of discrete data points extracted from a Euclidean correlation function. This challenge can only be overcome by introducing additional assumptions. For example, by treating the lattice "data" as analogous to experimental data, PDFs can be extracted from a fit to the lattice data by parametrising the functional form of the PDFs and fitting the resulting parameters [13–16]. This approach builds on the long-standing efforts to extract PDFs from a wide range of experimental processes [17], and is anchored by rigorous factorisation theorems.

Ab initio calculations of PDFs are in their infancy, but early indications suggest great promise. It is already clear that lattice data can be incorporated into the global fitting framework and will have the greatest impact where we have the least experimental data [18–20]. For example, lattice calculations of the strange quark PDF will shed light on the puzzling tensions between determinations from DIS and from W-boson production at the LHC [17, 21, 22]. Although these calculations are computationally challenging for lattice QCD,

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because of the need to calculate "disconnected contributions", theoretical input will certainly help clarify the strange content of the nucleon. The polarized gluon distribution provides another clear example: this calculation suffers from significant signal-to-noise issues, but there are no experimental data available, so even a moderate precision at moderate Bjorken-x will have a significant impact. The strange quark PDF and controlled determinations of the gluon distributions, are the highest priority short-term objectives for the community. More ambitiously, but of direct importance at the EIC, lattice calculations may provide an opportunity to study small-x physics from first principles, although this will undoubtedly require exascale computing resources coupled with improved theoretical methods.

Global fits of GPDs Given the promising results from lattice determinations of PDFs, the question naturally arises: can lattice QCD contribute to global efforts to extract GPDs from experimental data, and if so, how? Generalized form factors and other Mellin moments of GPDs are already accessible to lattice QCD (see, for example, the recent results in [23, 24]) and will certainly help constrain global fits. In principle, there are two clear reasons that one should expect lattice determinations of the x dependence of GPDs to have an even greater impact. First, conceptually (if not practically) GPDs are a straightforward extension of the existing methods for determining PDFs from lattice QCD to off-forward kinematics (which, in practice, entails only that the source and sink interpolating hadron operators have different momenta). Second, there are very few experimental data on GPDs, at least in part because of the challenge of extracting GPDs from experimental signals (for example, to extract GPDs from deeply virtual Compton scattering, one must first disentangle effects of the competing Bethe-Heitler process). Global fits are in their infancy [25–27], and thus there is a golden opportunity for lattice calculations to contribute to this effort

This is all very well in theory, but in reality, there is a long road to global fits of GPDs that incorporate lattice data. On the computational side, nonzero momentum transfer exacerbates many of the challenges associated with extracting x-dependent quantities, particularly the exponential noise-to-signal issue that plagues calculations of baryons and the computational cost associated with working in the Breit frame. Moreover, the greater number of degrees of freedom dramatically increases the computational cost, because individual numerical calculations must be carried out for every kinematic point. The first proof-of-principle results are therefore currently restricted to a small kinematic region close to the forward limit [28, 29]. On the theoretical side, there is a still a need for the development of models and physically-motivated parametrisations and for continued formal developments associated with global fits.

Questions for the community Global fits of GPDs are therefore an ambitious program that will span the next decade or so, and will require the combined efforts of lattice theorists, phenomenologists and experimentalists. First, the lattice community must map out the kinematic reach of lattice calculations and the corresponding systematic uncertainties associated with different kinematic regions, for different GPDs. Second, the structure community needs to understand how the results will compare to, or complement, both existing results and planned experiments at JLab. Third, prioritising the most desirable lattice calculations, or those that will have greatest impact, will help focus effort and resources on key quantities, because these calculations will be computationally very expensive. Finally, concerted efforts to draw these elements together will be necessary for lattice QCD to help guide the experimental program at the EIC.

Fostering links between phenomenologists, the global fitting and the lattice communities will be key to addressing these challenges. Two workshops, in Oxford [19] and Michigan [20], laid the groundwork for establishing connections between these groups and the new Center for Nuclear Femtography will build on those foundations. The need for collaboration only grows more pressing as the EIC era dawns and we continue to reveal the internal structure of the basic building blocks of the visible Universe.

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