The tensor renormalization group is poised for success

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Lattice regularization of quantum chromodynamics has been remarkably successful, both conceptually and practically, taking advantage of large-scale computing resources to explore the parameter space nonperturbatively. However, some of the most interesting regions of this parameter space are inaccessible using conventional Monte Carlo methods. This is due to sign problems in the sampling methods. It is also computationally expensive to sample large volumes at fine lattice spacing.

A promising and relatively-new numerical algorithm, called the tensor renormalization group (TRG), potentially has the ability to study these difficult-to-access regions of parameter space, and reach arbitrarily-large volumes [1, 2, 3, 4]. This is because the method does not rely on sampling using a probability weight, and the computational time scales like the logarithm of the volume. However, prior to a couple of years ago, the method was useful primarily in spacetime dimensions ≤ 2, due to its expensive computational cost in higher dimensions.

Excitingly, in the past couple of years there has been remarkable progress in the efficiency of TRG methods in higher dimensions [5, 6, 7, 8]. This includes simulations of a model with a sign problem, in four dimensions, with lattices up to size 1024^4 [7]. These approaches have made calculations in 2+1 and 3+1 dimensions completely feasible in terms of computational time and resources. Although this gain in efficiency came at the price of further truncation during the coarse-graining procedure, the fact that the TRG is immune to the sign problem, coupled with the advantage that large volumes are trivial using this approach, demands the further investigation of these higher-dimensional algorithms. If the loss in accuracy due to these truncations is negligible, or can be overcome through algorithmic tricks, the payoff when studying relevant physical models could be large. Moreover, if we apply ourselves to overcoming these accuracy problems, TRG methods could be useful in only a few years.

To assess the efficacy of these algorithms in higher dimensions, we should try various (toy and not toy) models, each with their own quirks and subtleties, to better understand the effects of truncation. This is an excellent opportunity to use the advanced computing facilities here in the United States in the spirit of the lattice quantum chromodynamics effort. Since the TRG is a large multi-linear algebra problem, it is quite amenable to parallelization. Moreover, often many of the symmetries of a model manifest themselves in the form of sparsity in the local tensors [9]. This sparsity is again conducive to efficient algorithms, and parallelization. Taking advantage of these opportunities at parallelization and optimization may already overcome whatever losses the truncations introduce in these higher-dimensional algorithms, and so, should be considered.

The TRG has the potential to make significant advances in our understanding of 3+1 dimensional quantum field theory. We can make rapid progress in fulfilling this potential by pushing the effort into developing efficient algorithms for high-performance computing, and studying a variety of physically relevant models. On top of this, the realization of this potential will prompt the fruitful collaboration between fields such as condensed matter, quantum information science, lattice gauge theory, and high energy theory/experiment.

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