Transverse-momentum-dependent parton distributions from lattice QCD

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An important goal of modern physics is to image the three-dimensional (3D) spatial and momentum structures of the proton, which will be vigorously pursued at collider experiments such as RHIC, JLab 12 GeV, as well as the future Electron-Ion Collider (EIC). The 3D momentum structure of the proton is encoded in the transverse-momentum-dependent parton distribution functions (TMDPDFs) which measure the longitudinal and transverse momentum of the quark and gluon partons. The TMDPDFs can be extracted through experimental processes such as semi-inclusive deep inelastic scattering (DIS) and Drell-Yan.

When the parton transverse momentum q_T is much larger compared to $\Lambda_{\rm QCD}$, the TMDPDF can be perturbatively matched onto the collinear PDFs, which has been extensively extracted through the global analysis of DIS and Drell-Yan data. This region is dominant in collisions where the measured transverse momentum is much larger than $\Lambda_{\rm QCD}$, such as at the LHC. In contrast, when $q_T \sim \Lambda_{\rm QCD}$, the TMDPDF is intrinsically nonperturbative and can only be determined through global analyses or first principle calculations. Moreover, the TMPDF depends on the energy of the parton, whose evolution is governed by the so-called rapidity anomalous dimension, or Collins-Soper kernel, which depends on q_T and also becomes nonperturbative when $q_T \sim \Lambda_{\rm QCD}$. As a result, there are more sources of uncertainties in the global analysis of TMDPDFs, and they are not well constrained from experiments compared to the PDFs. Therefore, a first principle calculation of the TMDPDF from nonperturbative QCD methods such as lattice gauge theory will significantly benefit the experimental programs in the foreseeable future.

Since the TMDPDFs are defined from correlation functions that depend on the real time, it is difficult to calculate them from the Euclidean lattice theory. Thanks to the development of the large-momentum effective theory (LaMET) [1–12] in recent years, it is now possible to obtain the full 3D information of TMDPDF from lattice QCD. The calculation includes a quasi-TMDPDF defined from equal-time correlators [3, 4, 6], as well as a soft function that can be obtained from a large-momentum-transfer pseudo-scalar meson form factor and its quasi-TMD wave function [8, 9] on the lattice. Moreover, by calculating the ratios of quasi-TMDPDFs at different hadron momenta, one can also extract the nonperturbative Collins-Soper kernel [5]. The first exploratory lattice calculation of the Collins-Soper kernel with the above method has been carried out in Refs. [13, 14], which shows promising sign of obtaining this observable for 0.2 GeV $< q_T < 1$ GeV with controlled precision. Besides, the soft function has also been calculated for the first time [15] using the method developed in Refs. [8, 9], where the Collins-Soper has also been extracted and shows agreement with that obtained in Ref. [14].

In this letter, we propose to calculate the nonperturbative TMDPDFs and the Collins-Soper kernel, which will provide useful inputs and constraints for their global analyses in current and future collider experiments. The efforts include:

1) Precision calculation of the Collins-Soper kernel in the nonperturbative region 0.2 GeV $< q_T < 1$ GeV, which can be directly used in the global analyses of TMPDFs;

2) Lattice calculation of the soft function. Understanding and improving the systematics in the calculation, which include operator mixing, lattice renormalization and power corrections;

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3) Lattice calculation the TMDPDF from the quasi-TMDPDF with the soft function. The short-term goal is to constrain the qualitative behavior of the TMDPDF at small transverse momentum, which can be used to constrain the phenomenological model in the global analyses. With increased resources in the future, the ultimate goal is to obtain a precise 3D image of the proton in momentum space.

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