Snowmass2021 - Letter of Interest

Lattice-QCD studies of inclusive B-meson decays

Snowmass Topical Groups: (check all that apply □/■)
■ (RF1) Weak decays of b and c quarks
■ (TF02) Effective field theory techniques
■ (TF05) Lattice gauge theory
■ (TF06) Theory techniques for precision physics
■ (TF07) Collider phenomenology
■ (CompF2) Theoretical Calculations and Simulation

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Abstract:
Long-standing discrepancies between inclusive and exclusive determinations of the CKM matrix elements V_{ub} and V_{cb} are one of the main physics drivers for ongoing experiments like Belle II and LHCb. Most theoretical calculations of exclusive decays use lattice gauge theory, while most inclusive calculations combine heavy quark effective theory and the operator product expansion. This letter describes prospects for lattice-QCD calculations of inclusive decays over the next decade, which are bright due to recent theoretical, algorithmic, and computing advances.

A long-standing puzzle in flavor physics is the discrepancy between inclusive and exclusive determinations of the CKM matrix elements V_{ub} and V_{cb}. This discrepancy is well-known and is one of the main physics drivers for ongoing experiments like Belle II and LHCb. A useful summary is given by the PDG review, *Semileptonic b-Hadron Decays, Determination of* V_{cb}, V_{ub}. [1].

Most calculations of exclusive decays use lattice gauge theory, while most inclusive calculations are done in the continuum combining heavy quark effective theory and the operator product expansion. Both methodologies are mature and report high precisions. To date, there are has not been much overlap between inclusive and exclusive calculations. To our knowledge, the only exceptions are lattice-QCD calculations of matrix elements which appear in both the spectroscopy of heavy mesons and in inclusive decay matrix elements (for example, Refs.[2, 3]). Given the fundamental importance the CKM matrix elements and of determining them consistently, it is critical to validate calculations using different theoretical techniques.

Prospects for lattice-QCD calculations of inclusive decays were recently given a boost with a pilot lattice study of an inclusive semileptonic decay by Gambino and Hashimoto [4], which builds on earlier work by Hashimoto (cf. Refs. [5, 6]). A clear exposition of the physics issues involved was given by Hansen, Meyer, and Robaina [7] and earlier papers [8, 9] describing calculations of real-time dynamics in QCD at finite temperature.

In the past, two closely related issues have complicated lattice calculations of inclusive quantities. First, physical scattering and decays take place in Minkowski space, while lattice calculations are done in Euclidean space. Second, lattice calculations are done in finite volume. As a result, in finite-volume decays there is no continuum of final states, just a sum of states with discrete momentum and energy. The recent theoretical insight is that this difficulty can be sidestepped by studying a decay rate smeared over energy.

Borrowing language and text from Ref. [7], the heart of the weak decay is the set of matrix elements

$$W_{\mu\nu}^{H_Q\to X}(v,q) = \frac{1}{2M_{H_Q}}\int d^4x\, e^{-iq\cdot x}\, \langle H_Q,\mathbf{p}|\mathcal{J}_\mu^\dagger(x)\mathcal{J}_\nu(0)|H_Q,\mathbf{p}\rangle\,, \tag{1}$$

where $\mathcal{J}_\mu$ is the flavor-changing current, M_{H_Q} is the mass of the decaying particle, the W-boson four-momentum is q^μ, and the four-velocity of the incoming hadron is $v^\mu = p_H^\mu/M_{H_Q}$. (In practice, the tensor would be decomposed into a sum of expressions with different tensor structure in v_μ and q_μ.) The task for lattice gauge theory is to compute the Euclidean-space finite-volume correlator of two currents between hadron states, produced using interpolating fields $\Psi_Q(\tau,\mathbf{p})$ at very early and very late times in the usual way:

$$\widetilde{G}_{\mu\nu,\mathbf{p}}^{H_Q\to X}(\tau,\mathbf{p}_x,L) \equiv 2E_{\mathbf{p}}L^3 e^{-E_{\mathbf{p}}\tau}\int d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \lim_{\substack{\tau_f\to\infty\\ \tau_i\to-\infty}} \frac{\langle\Psi_Q(\tau_f,\mathbf{p})\mathcal{J}_\mu^\dagger(\tau,\mathbf{x})\mathcal{J}_\nu(0)\Psi_Q^\dagger(\tau_i,\mathbf{p})\rangle_{\text{conn}}}{\langle\Psi_Q(\tau_f,\mathbf{p})\Psi_Q^\dagger(\tau_i,\mathbf{p})\rangle}\,, \tag{2}$$

where $E_{\mathbf{p}} = \sqrt{M_{H_Q}^2+\mathbf{p}^2}$. This object is used to build a Euclidean spectral function $\rho_{\mu\nu,\mathbf{p}}^{H_Q\to X}(\omega,\mathbf{p}_x,L)$ from

$$\widetilde{G}_{\mu\nu,\mathbf{p}}^{H_Q\to X}(\tau,\mathbf{p}_x,L) = \frac{1}{2\pi}\int_0^\infty d\omega\, e^{-\omega\tau}\rho_{\mu\nu,\mathbf{p}}^{H_Q\to X}(\omega,\mathbf{p}_x,L). \tag{3}$$

Extracting the physical spectral function ρ from G requires computing a numerically delicate inverse transformation. To regulate the inverse transformation, one approach defines a smoothed spectral function

$$\widehat{\rho}_{\mu\nu,\mathbf{p}}^{H_Q\to x}(p_x^0,\mathbf{p},L,\Delta) \equiv \int_0^\infty d\omega\; \widehat{\delta}_\Delta(p_x^0,\omega)\,\rho_{\mu\nu,\mathbf{p}}^{H_Q\to X}(\omega,\mathbf{p}_x,L), \tag{4}$$

where $\widehat{\delta}$ is a spread-out version of a delta function of width Δ. The hadronic tensor is recovered in the limit $\Delta\to 0$,

$$W_{\mu\nu}^{H_Q\to X}(p,q) = \frac{1}{2M_{H_Q}}\lim_{\Delta\to 0}\lim_{L\to\infty}\widehat{\rho}_{\mu\nu,\mathbf{p}}^{H_Q\to x}(p_x^0,\mathbf{p},L,\Delta) \tag{5}$$

Alternatively, one can stop at finite Δ and present a result for the smeared tensor. This smeared tensor can also be compared to smeared experimental data, as in the classic treatment by Poggio, Quinn, and Weinberg [10].

Any practical realization of these ideas must clear two technical hurdles. The first is the challenging calculation of the four-point function, Eq. (2). Modern techniques in computing all-to-all fermion propagators may help help ameliorate this cost [11]. The second difficulty is computing the smeared integral transform, for which at least three possible techniques are described in the literature. First, Ref. [4] employs a decomposition in terms of Chebyshev polynomials. Second, as sketched above, Refs. [7–9] propose using the Backus-Gilbert method, originally developed by geophysicists in the late 1960s. Third, Ref. [12] proposes using the so-called Bayes-Gauss-Fourier transform. A systematic comparison of the different techniques is essential to identify their respective strengths and weaknesses.

There is another possible line of attack on inclusive decays, which has not been discussed in the literature. The inclusive decay amplitude is quite similar to the amplitude for deep inelastic scattering, except that the gauge boson in the intermediate state has timelike rather than spacelike momentum. In both cases, the Minkowski-space amplitude is a matrix element of a product of two currents. The deep inelastic scattering matrix element is parameterized by structure functions or various parton distribution functions. There is already a large lattice-QCD literature calculating parton distribution functions (for a review, see Ref. [13]). The crucial ingredient is called a Ioffe-time distribution, which typical has the form:

$$M_\mu(n,P) = \left\langle P|\bar{\psi}(n)W(n,0)\Gamma_\mu\psi(0)|P\right\rangle . \tag{6}$$

Here n_μ is a four vector, $W(n,0)$ is a Wilson line reaching from $x=0$ to $x=n$, and $|P\rangle$ is a particle in a momentum eigenstate. Ordinary structure functions are defined for n_μ on the light cone through a Fourier transform of $M_\mu(n,P)$ while Euclidean lattice gauge theory provides these objects for spacelike n. Lattice-QCD calculations of a structure functions have many of the same issues as a calculation of an inclusive decay: a challenging matrix element (although it is a three-point function, not a four-point one) and an unstable Fourier transform. It might be that an alternate approach to inclusive decays could be found in the lattice-QCD literature on parton distribution functions.

Of course, besides the points already mentioned, inclusive decays also face the usual challenges common to all lattice-QCD calculations, e.g., controlling discretization errors, renormalization, and matching to the continuum theory. The methodology for controlling these systematic effects is well-understood and has been instrumental in establishing lattice QCD as a high-precision tool in the phenomenology of heavy quarks [14]. Of particular relevance for decays of B mesons is discretization effects associated with heavy quarks. Thanks to the usage of highly improved quark discretizations, it is now possible to simulate physical-mass b-quarks with fully relativistic actions [15]. This treatment greatly reduces systematic effects associated with renormalization and extrapolation to the physical point. With this experience and foreseeable computer power over the coming decade, we strongly believe that inclusive B-meons decays are a promising avenue of research in lattice gauge theory, supporting the Belle II and LHCb experimental programs.

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