Precision Resummation: Present and Future

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Abstract: The purpose of this planned review of precision resummation in high-energy physics is to provide an overview of recent advancements and promising directions in the field, with a particular attention to the phenomenological needs dictated by experiments at present and future accelerators. We plan to survey the field by analyzing various areas of interest (transverse momentum resummation, threshold resummation, event shapes, electroweak Sudakov, subleading power corrections, etc.) and by highlighting in which present and future experiments resummed results are relevant and which improvements are needed and expected.

1 Introduction

Extremely precise predictions for observables measured at particle accelerators are crucial in order to test the Standard Model (SM) of particle physics as well as to constrain the parameter space of proposed Beyond the SM scenarios. Precision physics is particularly important today since the long sought supersymmetric partners of SM particles – or any other clear sign of new physics – were not detected at the Large Hadron Collider (LHC).

Precise predictions for (differential) cross sections are obtained in fixed-order perturbation theory as expansions in power series in the SM coupling constants, in particular the strong coupling constant α_s, since the majority of processes studied at the LHC are mediated by the strong interaction. However, most phenomenologically relevant scattering processes involve a variety of mass scales, because of the kinematic constraints imposed on the phase space or because they involve massive particles. It is well known that in multi-scale processes the convergence of the perturbative series is spoiled by the presence of large logarithmic corrections that depend on the ratios of the scales involved in the process. Indeed, at each order in the expansion in powers of the coupling constant, higher powers of these logarithms can result in terms which are not negligible in comparison to terms of lower power in the coupling constant.

Resummation techniques make it possible to account for large logarithmic corrections to all orders in the coupling constant. This is achieved, roughly speaking, by exponentiating these corrections. This procedure can be justified and implemented in several ways, such as by examining the structure of the Feynman diagram involved in a given process, or by studying the process within an effective field theory approach. In particular, Soft-Collinear Effective Theory (SCET) is important in the context of collider physics. It allows for the systematic study of multi-scale processes and for the resummation of large corrections by means of renormalization group improved perturbation theory.

The purpose of the proposed review is to survey recent advancements in resummation studies, with a particular attention to the aspects of resummation that are expected to be phenomenologically relevant in the immediate and medium-term future. We plan to consider resummation based on both the SCET and the direct QCD approaches.

2 Covered Topics

We plan to organize the review in sections investigating the following areas of interest in the study of resummation

1) **Transverse momentum resummation** Transverse momentum spectra are an important hadron-collider precision observable. The per-mille level measurements for Z production, and percent level observations for many diboson processes from the LHC are a huge challenge for theory. We will review and compare existing resummation codes [1–19] and their underlying formalisms. The most accurate codes provide resummation up to N^{3}LL for single as well as diboson processes and also allow for the inclusion of fiducial cuts on the electroweak final-state particles. We will discuss challenges associated with power corrections [19, 20], non-perturbative effects and discuss the prospects to go beyond the few per-cent level predictions avaliable today.

2) **Threshold resummation** Fixed-order calculations beyond NLO are technically difficult. The resummation of soft gluon emission corrections allows for the evaluation of potentially large corrections to all orders. In LHC physics, soft-gluon resummation was initially carried out for processes with relatively simple final states, such as Higgs boson production and top-pair production. More recently, this approach was extended to the study of differential

distributions and finals states involving a larger number of massive particles. We plan to review soft-gluon resummation applied to the associated production of a top-antitop quark pair and a W/Z or Higgs boson, that was carried out up to NNLL accuracy, both by means of SCET techniques [21–24], and within the direct QCD approach [25–27]. The resummed calculations were combined to NLO QCD and EW corrections [28, 29] to obtain predictions that are now compared to early measurements of differential distributions [30].

3) **Event shapes** Event shape variables have a long history as precision observables at lepton colliders and some of them have been resummed to very high accuracy. However, there are interesting open questions, for example that the value of α_s extracted from thrust [31] is significantly lower than the value from other determinations [32]. At hadron colliders, these observables can provide new opportunities for improving our understanding of QCD, especially the study of factorizaton violation effects. Recently, NNLL resummation has been achieved for zero-jet [33–35], one-jet [36] and two-jet [37] production processes. Moreover, the three-loop order n-jettiness beam function was calculated in [38], which is an important ingredient in N^4LL resummation.

4) **Electroweak Sudakov** At high energy colliders, such as LHC and especially future 100 TeV hadron collider, the effect of electroweak corrections on the cross section can be significant. Therefore it is important to include electroweak Sudakov logarithms in the precision calculations. Electroweak contributions to PDF evolution have been computed recently in [39–42]. In addition, electroweak Sudakov resummation has been also extensively studied in the context of dark matter annihilation processes [43–50] due to its large impact on the annihilation cross sections.

5) **Jet observables, jet substracture methods, non-global logarithms** Jets not only display the behavior of QCD over a wide range of energy scales, but can also contain important signatures of exotic physics. In particular, recently jet substructure observables are playing a central role in a large number of analyses at the LHC. With this motivation, it is imperative to understand jet observables from first-principle QCD calculations. Some basic QCD resummation methods and a review of some recent developments for jet observables can be found in [51, 52]. In particular, a new effective theory for jet processes has been proposed within SCET framework [53, 54], where a multi-Wilson-line structure is introduced to capture the non-linear QCD evolution effects from the so-called *non-global logarithms*. Besides, in [55, 56] the Lund-jet plane has been proposed as a powerful tool to study jet substructures, where different types of single logarithms are systematically resummed.

6) **Subleading power corrections** With experiments requiring ever more precise predictions, it became necessary to study the structure of subleading power corrections both in QCD and SCET. The structure of subleding power corrections has been investigated to fixed order accuracy in the context of the method of regions and in SCET for different observables [57–67]. In addition, systematic derivations of factorization and resummation formulas at LL, up to next-to-leading power accuracy, were recently carried out [68–79]. In the review we will investigate the directions in which the study of subleading power corrections can evolve, as well as the theoretical challenges that it is currently facing.

From the point of view of phenomenology, we plan to focus on applications of resummation techniques that are relevant for the high-luminosity LHC, the proposed Electron Ion Collider (EIC), and a future e^+e^- collider that could operate as a Z or a Higgs factory. The review will deal with results published in the last five years, so that it complements and updates [80] and the review that can be found in the final chapter of [81].

References

[1] C. Balazs and C. Yuan, *Soft gluon effects on lepton pairs at hadron colliders*, *Phys. Rev. D* **56** (1997) 5558–5583 [hep-ph/9704258].

[2] G. Ladinsky and C. Yuan, *The Nonperturbative regime in QCD resummation for gauge boson production at hadron colliders*, *Phys. Rev. D* **50** (1994) 4239 [hep-ph/9311341].

[3] G. Bozzi, S. Catani, G. Ferrera, D. de Florian and M. Grazzini, *Production of Drell-Yan lepton pairs in hadron collisions: Transverse-momentum resummation at next-to-next-to-leading logarithmic accuracy*, *Phys. Lett. B* **696** (2011) 207–213 [1007.2351].

[4] T. Becher, M. Neubert and D. Wilhelm, *Electroweak Gauge-Boson Production at Small q_T: Infrared Safety from the Collinear Anomaly*, *JHEP* **02** (2012) 124 [1109.6027].

[5] D. de Florian, G. Ferrera, M. Grazzini and D. Tommasini, *Higgs boson production at the LHC: transverse momentum resummation effects in the $H \to 2\,\gamma$, $H \to WW \to l\nu l\nu$ and $H \to ZZ \to 4l$ decay modes*, *JHEP* **06** (2012) 132 [1203.6321].

[6] T. Becher, M. Neubert and D. Wilhelm, *Higgs-Boson Production at Small Transverse Momentum*, *JHEP* **05** (2013) 110 [1212.2621].

[7] J. Wang, C. S. Li, Z. Li, C. Yuan and H. T. Li, *Improved Resummation Prediction on Higgs Production at Hadron Colliders*, *Phys. Rev. D* **86** (2012) 094026 [1205.4311].

[8] M. Grazzini and H. Sargsyan, *Heavy-quark mass effects in Higgs boson production at the LHC*, *JHEP* **09** (2013) 129 [1306.4581].

[9] S. Catani, D. de Florian, G. Ferrera and M. Grazzini, *Vector boson production at hadron colliders: transverse-momentum resummation and leptonic decay*, *JHEP* **12** (2015) 047 [1507.06937].

[10] P. F. Monni, E. Re and P. Torrielli, *Higgs Transverse-Momentum Resummation in Direct Space*, *Phys. Rev. Lett.* **116** (2016), no. 24 242001 [1604.02191].

[11] W. Bizon, P. F. Monni, E. Re, L. Rottoli and P. Torrielli, *Momentum-space resummation for transverse observables and the Higgs $p_\perp$ at N^3LL+NNLO*, *JHEP* **02** (2018) 108 [1705.09127].

[12] F. Coradeschi and T. Cridge, *reSolve — A transverse momentum resummation tool*, *Comput. Phys. Commun.* **238** (2019) 262–294 [1711.02083].

[13] S. Camarda *et. al.*, *DYTurbo: fast predictions for Drell–Yan processes*, *Eur. Phys. J. C* **80** (2020), no. 3 251 [1910.07049]. [Erratum: Eur.Phys.J.C 80, 440 (2020)].

[14] V. Bertone, I. Scimemi and A. Vladimirov, *Extraction of unpolarized quark transverse momentum dependent parton distributions from Drell-Yan/Z-boson production*, *JHEP* **06** (2019) 028 [1902.08474].

[15] A. Bacchetta, V. Bertone, C. Bissolotti, G. Bozzi, F. Delcarro, F. Piacenza and M. Radici, *Transverse-momentum-dependent parton distributions up to N^3LL from Drell-Yan data*, *JHEP* **07** (2020) 117 [1912.07550].

[16] W. Bizon, A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, A. Huss, P. F. Monni, E. Re, L. Rottoli and D. M. Walker, *The transverse momentum spectrum of weak gauge bosons at N^3LL + NNLO*, *Eur. Phys. J. C* **79** (2019), no. 10 868 [1905.05171].

[17] S. Kallweit, E. Re, L. Rottoli and M. Wiesemann, *Accurate single- and double-differential resummation of colour-singlet processes with MATRIX+RadISH: W^+W^- production at the LHC*, 2004.07720.

[18] M. Wiesemann, L. Rottoli and P. Torrielli, *The $Z\gamma$ transverse-momentum spectrum at NNLO+N^3LL*, 2006.09338.

[19] M. A. Ebert, J. K. Michel, I. W. Stewart and F. J. Tackmann, *Drell-Yan q_T Resummation of Fiducial Power Corrections at N^3LL*, 2006.11382.

[20] M. A. Ebert and F. J. Tackmann, *Impact of isolation and fiducial cuts on q_T and N-jettiness subtractions*, *JHEP* **03** (2020) 158 [1911.08486].

[21] A. Broggio, A. Ferroglia, B. D. Pecjak, A. Signer and L. L. Yang, *Associated production of a top pair and a Higgs boson beyond NLO*, *JHEP* **03** (2016) 124 [1510.01914].

[22] A. Broggio, A. Ferroglia, B. D. Pecjak and L. L. Yang, *NNLL resummation for the associated production of a top pair and a Higgs boson at the LHC*, *JHEP* **02** (2017) 126 [1611.00049].

[23] A. Broggio, A. Ferroglia, G. Ossola and B. D. Pecjak, *Associated production of a top pair and a W boson at next-to-next-to-leading logarithmic accuracy*, *JHEP* **09** (2016) 089 [1607.05303].

[24] A. Broggio, A. Ferroglia, G. Ossola, B. D. Pecjak and R. D. Sameshima, *Associated production of a top pair and a Z boson at the LHC to NNLL accuracy*, *JHEP* **04** (2017) 105 [1702.00800].

[25] A. Kulesza, L. Motyka, T. Stebel and V. Theeuwes, *Soft gluon resummation for associated $t\bar{t}H$ production at the LHC*, *JHEP* **03** (2016) 065 [1509.02780].

[26] A. Kulesza, L. Motyka, T. Stebel and V. Theeuwes, *Associated $t\bar{t}H$ production at the LHC: Theoretical predictions at NLO+NNLL accuracy*, *Phys. Rev. D* **97** (2018), no. 11 114007 [1704.03363].

[27] A. Kulesza, L. Motyka, D. Schwartlander, T. Stebel and V. Theeuwes, *Associated production of a top quark pair with a heavy electroweak gauge boson at NLO+NNLL accuracy*, *Eur. Phys. J. C* **79** (2019), no. 3 249 [1812.08622].

[28] A. Broggio, A. Ferroglia, R. Frederix, D. Pagani, B. D. Pecjak and I. Tsinikos, *Top-quark pair hadroproduction in association with a heavy boson at NLO+NNLL including EW corrections*, *JHEP* **08** (2019) 039 [1907.04343].

[29] A. Kulesza, L. Motyka, D. Schwartlander, T. Stebel and V. Theeuwes, *Associated top quark pair production with a heavy boson: differential cross sections at NLO+NNLL accuracy*, *Eur. Phys. J. C* **80** (2020), no. 5 428 [2001.03031].

[30] **ATLAS** Collaboration, *Measurements of the inclusive and differential production cross sections of a top-quark-antiquark pair in association with a Z boson at $\sqrt{s} = 13$ TeV with the ATLAS detector*, .

[31] R. Abbate, M. Fickinger, A. H. Hoang, V. Mateu and I. W. Stewart, *Thrust at N^3LL with Power Corrections and a Precision Global Fit for $\alpha_s(m_Z)$*, *Phys. Rev. D* **83** (2011) 074021 [1006.3080].

[32] **Particle Data Group** Collaboration, P. Zyla et. al., *Review of Particle Physics*, *PTEP* **2020** (2020), no. 8 083C01.

[33] I. W. Stewart, F. J. Tackmann and W. J. Waalewijn, *The Beam Thrust Cross Section for Drell-Yan at NNLL Order*, *Phys. Rev. Lett.* **106** (2011) 032001 [1005.4060].

[34] T. Becher and X. Garcia i Tormo, *Factorization and resummation for transverse thrust*, *JHEP* **06** (2015) 071 [1502.04136].

[35] T. Becher, X. Garcia i Tormo and J. Piclum, *Next-to-next-to-leading logarithmic resummation for transverse thrust*, *Phys. Rev. D* **93** (2016), no. 5 054038 [1512.00022]. [Erratum: Phys.Rev.D 93, 079905 (2016)].

[36] T. T. Jouttenus, I. W. Stewart, F. J. Tackmann and W. J. Waalewijn, *Jet mass spectra in Higgs boson plus one jet at next-to-next-to-leading logarithmic order*, *Phys. Rev. D* **88** (2013), no. 5 054031 [1302.0846].

[37] A. Gao, H. T. Li, I. Moult and H. X. Zhu, *Precision QCD Event Shapes at Hadron Colliders: The Transverse Energy-Energy Correlator in the Back-to-Back Limit*, *Phys. Rev. Lett.* **123** (2019), no. 6 062001 [1901.04497].

[38] M. A. Ebert, B. Mistlberger and G. Vita, *N-jettiness beam functions at N^3LO*, 2006.03056.

[39] C. W. Bauer, N. Ferland and B. R. Webber, *Combining initial-state resummation with fixed-order calculations of electroweak corrections*, *JHEP* **04** (2018) 125 [1712.07147].

[40] C. W. Bauer, N. Ferland and B. R. Webber, *Standard Model Parton Distributions at Very High Energies*, *JHEP* **08** (2017) 036 [1703.08562].

[41] A. V. Manohar and W. J. Waalewijn, *Electroweak Logarithms in Inclusive Cross Sections*, *JHEP* **08** (2018) 137 [1802.08687].

[42] B. Fornal, A. V. Manohar and W. J. Waalewijn, *Electroweak Gauge Boson Parton Distribution Functions*, *JHEP* **05** (2018) 106 [1803.06347].

[43] M. Baumgart, I. Z. Rothstein and V. Vaidya, *Calculating the Annihilation Rate of Weakly Interacting Massive Particles*, *Phys. Rev. Lett.* **114** (2015) 211301 [1409.4415].

[44] G. Ovanesyan, T. R. Slatyer and I. W. Stewart, *Heavy Dark Matter Annihilation from Effective Field Theory*, *Phys. Rev. Lett.* **114** (2015), no. 21 211302 [1409.8294].

[45] M. Bauer, T. Cohen, R. J. Hill and M. P. Solon, *Soft Collinear Effective Theory for Heavy WIMP Annihilation*, *JHEP* **01** (2015) 099 [1409.7392].

[46] G. Ovanesyan, N. L. Rodd, T. R. Slatyer and I. W. Stewart, *One-loop correction to heavy dark matter annihilation*, *Phys. Rev. D* **95** (2017), no. 5 055001 [1612.04814]. [Erratum: Phys.Rev.D 100, 119901 (2019)].

[47] M. Baumgart, T. Cohen, I. Moult, N. L. Rodd, T. R. Slatyer, M. P. Solon, I. W. Stewart and V. Vaidya, *Resummed Photon Spectra for WIMP Annihilation*, *JHEP* **03** (2018) 117 [1712.07656].

[48] M. Beneke, A. Broggio, C. Hasner and M. Vollmann, *Energetic γ-rays from TeV scale dark matter annihilation resummed*, *Phys. Lett. B* **786** (2018) 347–354 [1805.07367].

[49] M. Beneke, A. Broggio, C. Hasner, K. Urban and M. Vollmann, *Resummed photon spectrum from dark matter annihilation for intermediate and narrow energy resolution*, *JHEP* **08** (2019) 103 [1903.08702]. [Erratum: JHEP 07, 145 (2020)].

[50] M. Beneke, C. Hasner, K. Urban and M. Vollmann, *Precise yield of high-energy photons from Higgsino dark matter annihilation*, *JHEP* **03** (2020) 030 [1912.02034].

[51] A. J. Larkoski, I. Moult and B. Nachman, *Jet Substructure at the Large Hadron Collider: A Review of Recent Advances in Theory and Machine Learning*, *Phys. Rept.* **841** (2020) 1–63 [1709.04464].

[52] S. Marzani, G. Soyez and M. Spannowsky, *Looking inside jets: an introduction to jet substructure and boosted-object phenomenology*, vol. 958. Springer, 2019.

[53] T. Becher, M. Neubert, L. Rothen and D. Y. Shao, *Effective Field Theory for Jet Processes*, *Phys. Rev. Lett.* **116** (2016), no. 19 192001 [1508.06645].

[54] T. Becher, M. Neubert, L. Rothen and D. Y. Shao, *Factorization and Resummation for Jet Processes*, *JHEP* **11** (2016) 019 [1605.02737]. [Erratum: JHEP 05, 154 (2017)].

[55] F. A. Dreyer, G. P. Salam and G. Soyez, *The Lund Jet Plane*, *JHEP* **12** (2018) 064 [1807.04758].

[56] A. Lifson, G. P. Salam and G. Soyez, *Calculating the primary Lund Jet Plane density*, 2007.06578.

[57] E. Laenen, G. Stavenga and C. D. White, *Path integral approach to eikonal and next-to-eikonal exponentiation*, *JHEP* **03** (2009) 054 [0811.2067].

[58] E. Laenen, L. Magnea and G. Stavenga, *On next-to-eikonal corrections to threshold resummation for the Drell-Yan and DIS cross sections*, *Phys. Lett. B* **669** (2008) 173–179 [0807.4412].

[59] D. Bonocore, E. Laenen, L. Magnea, L. Vernazza and C. D. White, *The method of regions and next-to-soft corrections in Drell–Yan production*, *Phys. Lett. B* **742** (2015) 375–382 [1410.6406].

[60] D. Bonocore, E. Laenen, L. Magnea, L. Vernazza and C. White, *Non-abelian factorisation for next-to-leading-power threshold logarithms*, *JHEP* **12** (2016) 121 [1610.06842].

[61] I. Moult, L. Rothen, I. W. Stewart, F. J. Tackmann and H. X. Zhu, *Subleading Power Corrections for N-Jettiness Subtractions*, *Phys. Rev. D* **95** (2017), no. 7 074023 [1612.00450].

[62] I. Moult, L. Rothen, I. W. Stewart, F. J. Tackmann and H. X. Zhu, *N -jettiness subtractions for $gg \to H$ at subleading power*, *Phys. Rev. D* **97** (2018), no. 1 014013 [1710.03227].

[63] M. A. Ebert, I. Moult, I. W. Stewart, F. J. Tackmann, G. Vita and H. X. Zhu, *Power Corrections for N-Jettiness Subtractions at $\mathcal{O}(\alpha_s)$*, *JHEP* **12** (2018) 084 [1807.10764].

[64] R. Boughezal, A. Isgr and F. Petriello, *Next-to-leading-logarithmic power corrections for N-jettiness subtraction in color-singlet production*, *Phys. Rev. D* **97** (2018), no. 7 076006 [1802.00456].

[65] R. Boughezal, A. Isgr and F. Petriello, *Next-to-leading power corrections to $V + 1$ jet production in N-jettiness subtraction*, *Phys. Rev. D* **101** (2020), no. 1 016005 [1907.12213].

[66] M. A. Ebert, I. Moult, I. W. Stewart, F. J. Tackmann, G. Vita and H. X. Zhu, *Subleading power rapidity divergences and power corrections for q_T*, *JHEP* **04** (2019) 123 [1812.08189].

[67] L. Cieri, C. Oleari and M. Rocco, *Higher-order power corrections in a transverse-momentum cut for colour-singlet production at NLO*, *Eur. Phys. J. C* **79** (2019), no. 10 852 [1906.09044].

[68] D. Bonocore, E. Laenen, L. Magnea, S. Melville, L. Vernazza and C. White, *A factorization approach to next-to-leading-power threshold logarithms*, *JHEP* **06** (2015) 008 [1503.05156].

[69] M. Beneke, M. Garny, R. Szafron and J. Wang, *Anomalous dimension of subleading-power N-jet operators*, *JHEP* **03** (2018) 001 [1712.04416].

[70] M. Beneke, M. Garny, R. Szafron and J. Wang, *Anomalous dimension of subleading-power N-jet operators. Part II*, *JHEP* **11** (2018) 112 [1808.04742].

[71] I. Moult, I. W. Stewart, G. Vita and H. X. Zhu, *First Subleading Power Resummation for Event Shapes*, *JHEP* **08** (2018) 013 [1804.04665].

[72] M. Beneke, A. Broggio, M. Garny, S. Jaskiewicz, R. Szafron, L. Vernazza and J. Wang, *Leading-logarithmic threshold resummation of the Drell-Yan process at next-to-leading power*, *JHEP* **03** (2019) 043 [1809.10631].

[73] N. Bahjat-Abbas, D. Bonocore, J. Sinninghe Damste, E. Laenen, L. Magnea, L. Vernazza and C. White, *Diagrammatic resummation of leading-logarithmic threshold effects at next-to-leading power*, *JHEP* **11** (2019) 002 [1905.13710].

[74] M. Beneke, M. Garny, S. Jaskiewicz, R. Szafron, L. Vernazza and J. Wang, *Leading-logarithmic threshold resummation of Higgs production in gluon fusion at next-to-leading power*, *JHEP* **01** (2020) 094 [1910.12685].

[75] I. Moult, I. W. Stewart, G. Vita and H. X. Zhu, *The Soft Quark Sudakov*, *JHEP* **05** (2020) 089 [1910.14038].

[76] M. Beneke, A. Broggio, S. Jaskiewicz and L. Vernazza, *Threshold factorization of the Drell-Yan process at next-to-leading power*, *JHEP* **07** (2020) 078 [1912.01585].

[77] Z. L. Liu and M. Neubert, *Factorization at subleading power and endpoint-divergent convolutions in $h \to \gamma\gamma$ decay*, *JHEP* **04** (2020) 033 [1912.08818].

[78] Z. L. Liu and M. Neubert, *Two-Loop Radiative Jet Function for Exclusive B-Meson and Higgs Decays*, *JHEP* **06** (2020) 060 [2003.03393].

[79] Z. L. Liu, B. Mecaj, M. Neubert, X. Wang and S. Fleming, *Renormalization and Scale Evolution of the Soft-Quark Soft Function*, *JHEP* **07** (2020) 104 [2005.03013].

[80] G. Luisoni and S. Marzani, *QCD resummation for hadronic final states*, *J. Phys. G* **42** (2015), no. 10 103101 [1505.04084].

[81] T. Becher, A. Broggio and A. Ferroglia, *Introduction to Soft-Collinear Effective Theory*, vol. 896. Springer, 2015.