# Theoretical developments in the SMEFT at dimension-8 and beyond 

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## INTRODUCTION

The Standard Model (SM) has so far been remarkably successful in describing all data coming from both lowenergy experiments and high-energy colliders. Although the search for new particles is continuing, it is becoming increasingly important to search for potentially small and subtle indirect signatures of new physics. A convenient theoretical framework for performing such searches when only the SM particles are known is the SM effective field theory (SMEFT) which contains higher-dimensional operators formed from SM fields. The SMEFT is an expansion in an energy scale $\Lambda$ at which the effective theory breaks down and new fields must be added to the Lagrangian. The leading dimension-6 operators characterizing lepton-number conserving deviations from the SM have been classified for some time now [1-3].

Less is known about terms at dimension- 8 and beyond in the SMEFT expansion. The number of operators at each order in the expansion has been determined [4], and ideas have been developed on how to systematically derive the structure of these operators [5]. Re-
cently the complete dimension-8 SMEFT basis was constructed $[6,7]$. More recently the complete dimension-9 SMEFT basis was constructed as well [8, 9].

It is our goal in this contribution to review the status of the SMEFT at dimension-8 and beyond. We will discuss the advances that have led to a complete construction of the operator basis at dimension- 8 , and the counting of operators to all orders in $1 / \Lambda$. We will present examples where all-orders results in the $1 / \Lambda$ expansion have been obtained. In several cases novel phenomenological consequences appear first at dimension- 8 or beyond. We will review such examples in this article.

## OPERATOR COUNTING BEYOND DIMENSION-6

The Hilbert series approach has been established to systematically enumerate EFT operators that are subject to redundancies (symmetry group, integration by parts, equation of motion), and was applied to the SMEFT in [4]. Hilbert series are akin to partition or generating functions, containing detailed information about the
number of operators with a given field content. They can provide valuable input to operator construction at dimension-8 and beyond. A breakdown of operators into those that are parity $(P)$ even and $P$-odd can be systematically accounted for using Hilbert series methods [10]. In the white paper we will review these techniques and how to further account for charge conjugation $(C)$, so as e.g. to enable systematic identification of $C P$-odd operators at dimension- 8 and above, which could have particularly striking signatures.

## CONSTRUCTION OF THE DIMENSION-8 OPERATOR BASIS

A prerequisite to studying the phenomenology of dimension-8 operators in a consistent fashion is the construction of a complete basis of dimension- 8 operators. The classification of dimension- 8 operators is greatly aided by the fact that the number of each type of operator is known via the Hilbert series approach [4], and the classification builds upon the systematic techniques developed in Refs. [3,5] for determining the explicit forms of the operators. In the white paper that will accompany this LOI we will review the construction of the dimension8 SMEFT basis with an emphasis on aspects of the construction that are most germane to bases at $d>6$.

## ALL-ORDERS RESULTS IN THE $1 / \Lambda$ EXPANSION

While the number of operators at $d>6$ grows rapidly, it is possible to use integration by parts and equation of motion redundancies to define a basis where the number of operators that contribute to 2 - and 3 -point vertices is small and is approximately constant at each mass dimension. Using this basis, 2- and 3-point vertices -which control the the physics of $1 \rightarrow 2$ decays, resonant $2 \rightarrow 2$ scattering, and the mapping of electroweak inputs into Lagrangian parameters - can be defined to all orders in $1 / \Lambda$, where $\Lambda$ is the scale suppressing higher dimensional operators. In the white paper accompanying this LOI, we will review how the compact form allowed by this 'geometric' basis [11, 12] can be used to study $1 \rightarrow 2$ process at $\mathcal{O}\left(1 / \Lambda^{4}\right)$. The truncation error from higher orders in $1 / \Lambda$ is a key ingredient in SMEFT global fits, and explicit $\mathcal{O}\left(1 / \Lambda^{4}\right)$ results are helpful to understand the size and behavior of this uncertainty.

## ON-SHELL APPROACH TO THE SMEFT

On-shell amplitude techniques provide an alternative approach to the SMEFT, notably avoiding the gauge
and field-redefinition redundancies inherent in the Lagrangian treatment. This is particularly useful to explore patterns and properties arising from scattering amplitudes in the presence of higher-dimensional operators. For example, the structure of the anomalous dimension matrix $[13-17]$ as well as non-interference theorems are made manifest in this approach [18-20]. On the other hand, the direct construction of on-shell amplitudes, for either massless or massive particles [21-24] can for instance replace the enumeration of operators through the Hilbert series. An important development in this area is the recent little-group-covariant formalism of [25] which has allowed for pushing the on-shell techniques also for massive and arbitrary spin external particles. The renormalizable SM amplitudes were studied in [26-28] and the map between the massive three-point on-shell amplitudes to dimension-six operators in the Warsaw basis were presented in [29, 30]. A further step was taken in [30], where the electroweak symmetry is not built-in but can be recovered by imposing perturbative unitarity. Moreover, a systematic construction of three and four-point nonfactorizable amplitudes were presented in [31]. This approach also yields all-order results in $v / \Lambda$, whose powers are all absorbed in constant amplitude coefficients. Treelevel recursion relations for massive amplitudes have been investigated too $[32,33]$. The development of this alternative view on the SMEFT provides new insight and allows for new computations.

## NOVEL PHENOMENOLOGICAL CONSEQUENCES AT DIMENSION-8

The naive expectation is that deviations from the SM induced by dimension- 8 operators are subdominant to dimension-6 deviations and can be safely ignored. This is indeed often the case, although the increasing precision of LHC data will eventually require the inclusion of even such subleading effects in global fits. However, there are interesting cases where the dimension- 8 terms are sometimes the leading contributions to observables due to symmetry considerations or the structure of the corresponding SM amplitudes. In such cases it is important to quantify their effects in order to guide experimental searches. Such probes may also serve as smoking-gun signatures of dimension-8 extensions of the SM.

In the white paper accompanying this LOI we will review examples where dimension-8 effects give qualitatively different results than dimension-6. For example, a class of dimension- 8 operators in the SMEFT generate novel angular dependences in Drell-Yan lepton-pair production not accounted for in current experimental analyses [34]. They are not generated at leading-order by dimension-6 operators in the SMEFT, nor by QCD effects in the SM. This offers the possibility of extending the current experimental studies to search for this po-
tential smoking－gun signature of new physics appearing through dimension－8 effects．

We will further discuss a few examples of characteris－ tic manifestations of dimension－ 8 operators in low－energy precision experiments．We will study the modifications to $\beta$ decay rates and spectra induced by the dimension－ 8 operators that generate new angular distributions in charged－current Drell－Yan，thus identifying possible dis－ tinctive low－energy signatures，and address the impact of dimension－8 operators on low－energy constraints on non－ standard charged－current interactions．At dimension－8，a new class of flavor diagonal CP－odd operators that break time－reversal，but not parity，arises．These generate new $T$－odd $P$－even effects at low energy，including toroidal quadrupole moments of particles with angular momen－ tum greater than one（such as the deuteron or positro－ nium）$[35,36]$ and $T$－violating asymmetries in proton－ deuteron scattering［37］．We will examine the sensitivity of these experiments，and the implications for colliders． Additionally，we will provide examples of matching of the SMEFT at dimension－8 to other effective field theories， which are relevant for the hints of lepton universality vi－ olation in the semi－leptonic decays of $B$ mesons．

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