Improvements in parton shower algorithms

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I. INTRODUCTION

Parton shower event generators have proven to be very important since their introduction in the 1980s [1, 2]. Discussion of these tools and of prospects for their improvement is included in the program of the Energy Frontier. We suggest that a complementary discussion should be part of the Precision branch of the Theory Frontier.

The focus within the Theory Frontier would be on the algorithms used to generate a simulated parton shower. These algorithms are based on a detailed understanding of the structure of the QCD. In recent years there has been substantial work to translate our knowledge of this structure into practical computer algorithms. This work is closely related to developments in extending perturbative calculations of important hard processes to higher perturbative order. The work is also closely related to work to rearrange QCD perturbation theory for processes that contain large logarithms in their perturbative expansion. One wants to sum the large logarithms so as to improve the precision of predictions. The idea of including this discussion within the Theory Frontier is to emphasize that there is a lot of theoretical work that goes into developing these essential tools.

II. ITEMS FOR DISCUSSION

We list below some of the topics that could be part of the Theory Frontier discussion.

1. *Matching.* A parton shower is initiated by a hard scattering process, say $gg \to \text{Higgs}$. If the hard scattering is calculated at lowest order, then the result is conceptually simple: the parton shower provides an approximated version of higher order corrections. However, if we want to use a hard process calculated at, say, NLO, then we need to subtract the approximated NLO corrections generated by the shower. There has been a large amount of work on this in recent years and work is ongoing [3–22].

2. *Merging.* We may want to consider together two different processes, each calculated at beyond lowest order (for instance $gg \to \text{Higgs}$ and $gg \to \text{Higgs} + \text{jet}$). Then each hard process can initiate a parton shower. Evidently, there is a certain amount of ambiguity in exactly how these processes should be combined. Again, there has been a large amount of work on this in recent years and work is ongoing [23–25].

3. *NLO shower.* Current parton shower algorithms are based on parton splitting probabilities calculated at lowest order, order α_s^1. One might hope to have a parton shower based on parton splitting probabilities calculated at order α_s^2. There has been some recent progress in this direction [26–30]. We have provided a general framework that we think could guide the construction of an NLO shower [31]. Although we believe that a complete algorithm lies some years in the future, we believe that the effort is important.

4. *Quantum interference.* Parton shower algorithms need to respect quantum mechanics. That is, one should consider the evolution of the quantum amplitude. One of the earliest shower algorithms, the one in Herwig, was invented to do just this in an approximate way. Most modern parton showers are of the "dipole" sort, so that one includes (with approximations) emissions from both members of the dipole, including the interference between emission from one member of a dipole and emission from the other. In this way, it is the quantum amplitudes that evolve.

5. *Color.* Partons carry quantum SU(3) color. This means that there is a bra amplitude describing many partons with their color and a ket amplitude for many partons with their color. Gluon emissions change the color state, as do virtual gluon exchanges. This is not so easy to describe in a computer because the color space for, say, twenty partons has approximately 10^{36} dimensions. Furthermore, an approximate version of virtual diagrams is included in a parton shower as an exponential, the Sudakov exponential. One can, of course, exponentiate a matrix on a computer, but not in 10^{36}

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dimensions.

Most programs use what is called the leading color (LC) approximation, which gives the leading term in an expansion in powers of $1/N_c^2$ (with $N_c = 3$). There is an improved version called the LC+ approximation [32], but this is still a rather crude approximation. Work to do better is currently ongoing. One method expands perturbatively in the difference between the full color splitting functions and their LC+ approximation [33]. A very recent, independent, algorithm [34] is more limited in its current capabilities but is rather similar in its approach.

6. *Spin.* Partons carry quantum spin. The spin of gluons affects the dependence of their splitting functions on the azimuthal angle of their decays. Thus one should keep track of quantum spin in the parton amplitudes. The alternative is to average over spins at each step, thus losing information that affects azimuthal angle distributions. For certain technical reasons, the implementation of quantum spin in a parton shower algorithm is much easier than the implementation of color. However, the needed implementation has, for the most part, not been realized. The notable exception is HERWIG7 [35]. We can anticipate that more development in this direction will take place in the future.

7. *Summation of large logarithms.* Many cross sections that play a role in particle physics depend on two very different momentum scales. In consequence, the coefficient of α_s^n in the perturbative expansion of such a cross section will contain powers of the logarithm of L of the ratio of these scales. Typically, we find contributions proportional to $\alpha_s^n L^{2n-1}$ or $\alpha_s^n L^{2n}$. An example is the cross section to produce a virtual photon with squared momentum Q^2 and with transverse momentum k_T, with $L = \log(Q^2/k_T^2)$. The large logarithms L spoil the usefulness of fixed order perturbation theory in calculating the cross section. There has been a very substantial theoretical effort over the years to sum the perturbative contributions that contain the most powers of L. For instance, soft-collinear-effective theory (SCET) has often been used for this purpose in recent years.

   This sort of analytical large logarithm summation is adapted to a particular observable cross section. On the other hand, a parton shower event generator samples many simulated events and allows the user to measure *any* cross section involving the resulting partons (or hadrons if one applies a hadronization model). Thus a parton shower is much more flexible than a dedicated calculation of the same cross section. Furthermore, a parton shower uses parton splitting functions that contain the soft and collinear singularities of QCD, so it has the potential to sum the large logarithms correctly.
   Does it? In some cases, it does [36, 37]. However, the answer depends on what the details of the parton shower are and what logarithms one would like to sum. Clearly, it is important to understand this connection better. There has been interesting recent work on this subject [38] and we can anticipate more results in the near future.

8. *Threshold logarithms.* There is one class of large logarithms that is typically not included in parton shower algorithms. These are the "threshold logarithms" that occur in hard scattering cross sections at hadron colliders [39]. These logarithms can be thought of as arising from a mismatch between the kinematic limits in the DGLAP evolution equation for parton distribution functions and the kinematic limits of splittings in a parton shower or in the contributions to the theoretical cross section beyond leading order. The effects of threshold logarithms are often important, so there has been an extensive effort over the years to analyze them analytically.

   The effect of threshold logarithms can be incorporated in a parton shower algorithm [40, 41]. Typically, this effect is left out of parton shower event generators, although the first perturbative term in the threshold log summation is included if the parton shower is matched to an NLO perturbative calculation of the hard scattering cross section. One can anticipate that a threshold factor will be included in more parton shower event generators so as to improve their accuracy in the future.

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