Theory Needs for FCC-ee Part II: New methods for SM calculations

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Abstract

The FCC-ee is at the High Energy and Precision Frontier and will provide a set of ground-breaking measurements of a large number of new-physics sensitive observables, with improvement by one to two orders of magnitude in experimental precision. The full exploitation of the significantly increased experimental precision in Z-pole observables, W boson and top quark masses, and a broad array of Higgs observables, necessitates SM predictions accurate at a level commensurate with this precision. In this submission we outline the challenges posed for the necessary development of theoretical methods required for the calculation of higher order EWPO calculations in the SM, both for electroweak and QCD sectors, at a level commensurate with the experimental precision.

The FCC-ee would realise an exceptionally clean experimental set-up, as compared to hadron colliders, hence the demands in precision are much higher, requiring leaps in theoretical computations for Standard Model phenomena and electroweak pseudo-observables (EWPOs). Examples include partial widths and couplings of Z and W, forward-backward and polarization asymmetries, peak cross sections at the Z resonance, the effective electroweak mixing angle [1, 2], and additional EWPOs in the WW, ZH and $t\bar{t}$ processes. EWPOs encapsulate experimental data after extraction of well known and controllable QED and QCD effects, in a model-independent manner. They provide a convenient bridge between real data and the predictions of the SM. Contrary to raw experimental data (like differential cross sections), EWPOs are well suited for archiving and long term exploitation. In particular archived EWPOs can be exploited over long periods of time for comparisons with steadily improving theoretical calculations of the SM predictions, and for validations of new physics models beyond the SM. They are also useful for the comparison and combination of results from different experiments.

Note that for FCC-ee data analysis, due to the rise of non-factorisable QED effects above the experimental errors, Monte Carlo programs might become the standard for direct fitting of EWPOs to data, even at the Tera-Z stage. Section C3 of [2] describes possible forms of future EWPOs at FCC-ee experiments and required new MC software is specified. It is underlined there that due to non-factorisable QED contributions, the factorization of the multiphoton QED effects will have to be applied at the amplitude level. Additional quantities available in tau and heavy flavour physics will reach the 10^{-5} precision and are likely to require similar attention.

Also concerning the numerically important QED corrections, further refinements of factorization to infinite order of the multi-photon soft real and virtual contributions, and their resummation using Monte Carlo methods, have to be pursued in order to meet the accuracy needs. Again infra-red problems are crucial: The inclusion of collinear (mass) singularities must be incorporated in the existing schemes. Techniques for disentangling QED and EW corrections for two and more loops in the framework of soft photon factorization/resummation are in principle available, however their practical implementation will require more work. A central issue is the implementation of the higher loop terms in their proper form as Laurent series around the Z-boson pole, characterized by mass and width, in the scattering energy, which is mandated by analyticity, gauge-invariance and unitarity.

The high precision measurements of EWPOs at FCC-ee must be compared with theoretical predictions at least at the same level of accuracy and, since the theoretical SM calculation needed for FCC-ee is deeply intertwined with contemporary cutting-edge mathematical studies, this necessary experimental advance will engender novel innovations at the level of the calculational methods. This is because, in order to develop new computational methods to handle the anticipated pQFT calculations, a considerable synergy of pQFT with advanced and explorative mathematics, combined with progress in the computer algebra systems and algebraic geometry will be required to realise new cutting edge algorithms for analytical and numerical methods.

The level of complexity, at a given loop order, is defined by the number of virtual massive particles in the Feynman integrals. In one loop integrals the most complicated mathematical objects are Euler Dilogarithm functions. In last years it became obvious that analytical solutions to multiloop integrals, which are the ideal solutions to have, will involve elliptic functions, and will go beyond this class of special functions [3]. Recall that elliptic curves, modularity and modular forms were one of the key points to solve a long standing problem in mathematics, Fermat's Last Theorem [4].

The challenge outlined here goes beyond the NNLO level of the SM perturbative calculations. At the Tera-Z FCC-ee option we will need the leading electroweak 3-loop and QCD 4-loop contributions in order not to limit the interpretation of the Z resonance shape, and there are similar demands from the other FCC-ee modes. There is no closed form theory for perturbation theory calculations of Feynman integrals beyond one loop. For this reason, numerical integration methods are the most promising, if not the only, avenues for addressing those challenges. Analytical techniques are expected to be important in many respects, but numerical integration methods have advantages when increasing the number of masses and momentum scales. Fortunately, there has been impressive progress in recent years in this direction [2]. There are currently two numerical methods known to allow a systematic treatment of infra-red divergences. In 2014 the only advanced automatic numerical two-loop method was sector decomposition (SD). However, the corresponding software was not sufficiently developed to evaluate the complete set of Feynman integrals for the electro-weak bosonic two-loop corrections to the Z-boson decay with the desired high precision (up to 8 digits per integral). The task could be completed successfully with a substantial development of a competing numerical approach, based on Mellin-Barnes MB representations of Feynman integrals [5].These calculations are challenging due to the numerical role of particle masses M_Z, M_W, m_t, M_H, leading to (i) an enormous number of contributions, ranging from tens to hundreds of thousands of diagrams (at 3-loops), and (ii) the occurrence of up to four dimensionless parameters in Minkowskian kinematics (at $s = M_Z^2$) with intricate threshold and on-shell effects where contour deformation fails. To tackle more loops or legs, merging both the MB- and SD-methods in numerical calculations, is key for ultimate success.

A well founded expectation is that these two methods, combined with more specialized techniques for specific topologies, will enable the next steps of the necessary multi-loop calculations. There are also many opportunities for improvement in other approaches and software packages; see [2] for a recent overview. Presently there is lively activity in many areas of multi-loop calculations, and several promising (semi-) analytical methods are also under development [2]. For instance a treatment of multi-scale integrals beyond multiple polylogarithms, direct numerical calculations of Feynman integrals in $d = 4$, or calculations based on unitarity methods, all of them with first examples of two-loop numerical evaluations. The ability of the theory community to meet the challenges posed by the FCC-ee electroweak precision programme is demonstrated by the remarkable progress made in developing new calculational techniques and advances in the calculation of challenging higher-order calculations for hadronic processes. Results such as the N^{3}LO cross section for Higgs production and Drell-Yan processes, and the NNLO results for most 2→2 processes, were unthinkable just few years ago, and have been stimulated and subsequently realised as a result of the concrete needs of the LHC programme. Similarly, the challenge posed by FCC-ee precision measurements will stimulate significant progress in the particle physics theory community.

References

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