The N^{3}LO Frontier: Precision Predictions with QCD Perturbation Theory

Letter of Interest for Snowmass2021

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Perturbative QFT is the foundation of the exploration and interpretation of high energy particle collision experiments. It allows us to compare experimental observation on solid foundation to our state of the art understanding of fundamental interactions of nature. Matching or overcoming the precision achieved in experimental measurement in theoretical predictions is critical in order to improve our understanding of nature. Consequently, it is paramount to develop our capabilities to perform precise predictions far beyond the current state-of-the-art. In particular, it is crucial to make predictions at next-to-next-to-next-to leading order ($\mathrm{N^3LO}$) in QCD perturbation theory readily available for a large class of realistic observables in hadron collisions.

QFT perturbation theory is the tool that allows us to describe scattering processes that hold the key to understanding interactions of nature that are only accessible at the highest energies. Directly observing the remnants of scattering processes involving top quarks, electro-weak gauge bosons or the newly discovered Higgs boson are just a few examples. The Large Hadron Collider (LHC) will provide a wealth of data for such scattering processes in the years to come. Future facilities like the Electron Ion Collider (EIC) are in planning. The achieved precision in measurements at these facilities will severely challenge our capability to extract information from the experiment simply because we are limited by the precision of theoretical predictions.

In the past years, we have seen a large variety of essential LHC processes described at next-to-next-to leading order (NNLO) in QCD perturbation theory supplemented with electro-weak corrections at next-to leading order. In combination with state-of-the-art techniques for resummation of infrared sensitive observables and by incorporating perturbative predictions into realistic parton shower models we have begun the age of precision LHC observables. However, with the advent of the high luminosity phase of the LHC and the incredible wealth of collected data that is to come with it, it is obvious that determination of precision cross sections at the level of a few percent will not only be possible but a reality. In light of this reality theoretical precision for cross section prediction must improve and the truncation of the perturbative series of QCD at NNLO presents one of the mayor obstacles towards this.

The age of $\mathrm{N^3LO}$ QCD predictions for LHC observables began only recently with the computation of the production probability of the Higgs boson in hadron collisions [1–3]. Furthermore, by now inclusive cross sections for the Higgs boson production in vector boson fusion (VBF) in the DIS approach [4], bottom quark fusion [5, 6], for production of two Higgs bosons in gluon fusion [7] and VBF [8] and for the Drell-Yan production cross sections [9, 10] are available. These observables encapsulate already some of the key collider observables The picture that arises from studying the results of these computations is clear: $\mathrm{N^3LO}$ corrections for cross sections are universally at the order of a few percent and need to be controlled in order to achieve percent level precision for LHC observables. The obtained corrections stabilise the progression of the perturbative expansion and lead to a significantly improved description of the observables in question.

The path from idealised inclusive cross sections for a single colourless particle to more complex final states or realistic hadron collider observables that involve limitations on the momenta of observed particles is extremely challenging. Early steps towards this were achieved for the jet-veto cross section of gluon fusion Higgs production [11], for vector boson fusion Higgs boson cross sections [4] and for the rapidity distribution of the gluon fusion Higgs boson production cross section [12–14]. In the future, we should aim to make N^{3}LO the precision standard for highly energetic precision probes in hadron collider experiments.

Achieving N^{3}LO precision for a large range of LHC observables is an ambitious goal with a multitude of benefits. As a matter of fact the extraction of fundamental quantities like coupling constants and masses, the test of our understanding of the structure of interactions, the study of the components of hadrons and much more relies on it. The distinction of new physics from established Standard Model physics is only possible if we are able to predict the outcome of our experiments to sufficient precision. Beyond the phenomenological implications the tremendous theoretical challenge of improving our ability to predict will inspire us to improve our understanding of the structure of perturbative QFT. New and more efficient ways for theoretical computations and a deeper understanding of scattering of elementary particles are a consequence of this.

Achieving the goal of percent level physics in hadron collisions will require a concerted effort and support from particle physics phenomenology community. Below we identify only some of the mayor challenges that have to be overcome in the years to come.

1. **Accessibility and User Friendliness:** Creating frameworks that make N^{3}LO (and NNLO) predictions easily accessible for comparison to experimental data.
2. **Virtual Amplitudes:** New and improved techniques for the computation of multi-loop scattering amplitudes.
3. **Subtraction / Slicing Algorithms:** Efficient numerical schemes for the treatment of infrared singularities in radiative corrections at N^{3}LO.
4. **N^{3}LO PDFs:** Determining parton distribution functions at N^{3}LO in QCD perturbation theory.
5. **Mathematical Function Space:** Deepening our understanding of the mathematical structure of scattering amplitudes.
6. **Parton Showers:** Consistent combination of parton showers with fixed order perturbative computations at N^{3}LO.
7. **Resummation:** Complementing N^{3}LO computations and resummation techniques for infrared sensitive observables.
8. **Uncertainties:** Deriving / defining reliable uncertainty estimates for theoretical computations at the percent level.
9. **Beyond Leading Power Factorisation:** Exploring the limitations of leading power perturbative descriptions of hadron collision cross sections.

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