Multi-loop Amplitudes for Colliders

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Theory predictions at high perturbative orders enable precision phenomenology at high energy experiments. In this letter, we propose to survey recent analytic and computational advances for the calculation of multi-loop scattering amplitudes.

A standard workflow for fixed-order calculations involves symbolic manipulations to obtain scattering amplitudes in terms of master integrals, an analytic calculation of the master integrals in terms of standard mathematical functions, and a numerical integration over phase space using a scheme to handle infrared divergences. A major driver behind precision phenomenology is the ability to numerically evaluate multi-loop scattering amplitudes for the physical kinematics of interest.

For one-loop amplitudes, a high level of automation has been achieved using numerical methods to overcome limitations due to the increase in algebraic complexity with increasing numbers of legs and mass scales. More recently, significant progress has been made also at the multi-loop level through a better understanding of the underlying mathematical structure as well as through computational advances. Methods from polynomial ideal theory [1] allow for better control over the reduction of tensor integrals compared to classic integration-by-part identities. Furthermore, the usage finite field arithmetic [2–4] allows to bypass intermediate expression swell in what has traditionally been purely symbolic manipulations to construct the reduced amplitude. Another advance has been the extension of numerical unitarity techniques to multi-loop amplitude calculations [5–7].

At the multi-loop level, new processes typically require the calculation of unknown master integrals. Analytic calculations have the potential for very precise and fast numerical evaluations, but require a detailed understanding and algorithmic control over the special mathematical functions involved, such as multiple polylogarithms of algebraic arguments [8] or elliptic polylogarithms [9]. Meanwhile, purely numerical techniques can often provide answers with sufficient speed and precision for phenomenology when the analytical understanding still lags behind [4, 10, 11]. In all cases, a proper choice of basis can lead to great simplifications, e.g. a canonical basis [12] for the method of differential equations or a basis of finite integrals [13] for direct parametric integration. Expansion techniques offer precise numerical evaluations when differential equations are accessible but their exact analytic integration is challenging [14].

As a contribution to the Snowmass effort, we wish to survey different advances in perturbative methods for precision phenomenology. A special emphasis is on the computational demands for precise theory predictions for current and future measurements at colliders. In particular, we propose to characterize the methods and complexity of existing calculations to assess future computational demands, which are required to deliver results for the physics program of the LHC, EIC, HL-LHC and other future colliders.

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